Normal Rudiments of Arithmetic

Brooks

Christopher Sower Company
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Katherine E. Allen.

THE

NORMAL

RUDIMENTS OF ARITHMETIC,

ORAL AND WRITTEN,

DESIGNED FOR

PRIMARY AND INTERMEDIATE CLASSES IN PUBLIC AND PRIVATE SCHOOLS.

BY

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"The highest science is the greatest simplicity."

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PREFACE.

THIS little work is intended for the child's first lessons in Arithmetic. It is based on a careful study of the natural development of the numerical idea in children, and aims to aid the teacher in giving this development. Some of its leading peculiarities will be briefly stated.

SPECIAL FEATURES.—Attention is first called to the following special features which distinguish this work:

1. The first object in the course of instruction is to develop the idea of number with children. This idea arises in the mind in connection with objects. A child sees the objects and thinks the number. All primary instruction is therefore to be given in the concrete. This is made a prominent feature of the work.

2. Objects and numbers, however, are not identical. One is a thing of sense, the other is an idea of the mind. This distinction, often lost sight of by modern educators, should be clearly seen by the teacher. The attempt to hold the mind of the pupil down to objects and explain all numerical processes by means of blocks and sticks would be to enfeeble its powers and give it a wrong idea of numbers. The aim has been, therefore, to gradually lift the mind up to the conception of number in the abstract.

3. The basis of Arithmetic consists of the fundamental processes of Addition, Subtraction, Multiplication, and Division. These operations are based on what may be called "elementary results," by which is meant the elementary sums and differences as far as 9 and 9 are 18, and the elementary products as far as "9 times 9" with the corresponding quotients. These elementary results constitute the alphabet of Arithmetic, and are to be committed to memory. Their extent is limited by our decimal system of numeration and notation to nine, though custom has fixed the limit at twelve.

4. These elementary processes are to be taught in their relation to one another. Addition and Subtraction are the primary processes, and are the converse of each other. They are therefore to be taught together in deriving the elementary results. Multiplication is a

special process of Addition, and should therefore be so taught as to exhibit this relation. Division is the converse of Multiplication, and is presented in this relation rather than as an independent process. These four processes are to be combined in elementary instruction in as close a relation as the natural development of the mind of a child will permit.

5. The ideas of these elementary processes can be developed along with the ideas of numbers. In the development of these ideas it has been found convenient to begin with the first numbers and treat them in regular order as far as *twenty*. This will give all the elementary results of addition and subtraction, and several of the elementary products and quotients. The first section of the book therefore treats of numbers from *one* to *twenty*. In deriving the quotients up to this limit no inexact divisions are required, the object being merely to reverse the elementary products.

6. After reaching the number twenty, instead of continuing the numbers in order to 100 or 144, as in the Grube system, the order of the elementary products is followed, the object being to have the pupils derive and commit the elementary products and quotients. These exercises constitute the second section of the work.

7. In connection with these exercises the elements of fractions and denominate numbers are incidentally introduced, so that by the time pupils have reached the formal treatment of these subjects a solid foundation has been laid for them.

8. During all these exercises in becoming familiar with numerical ideas and the elementary results, pupils are gradually taught to write numbers according to the Arabic system. The elements of the fundamental operations with numbers exceeding the elementary results are also gradually introduced as preparatory to the formal treatment of them.

9. Having laid this foundation of the science and the art in the mind of the learner, the subject is taken up in regular order as indicated by the science under the heads of Numeration and Notation, Fundamental Operations, Fractions, Decimals, Denominate Numbers, etc.

General Features.—In addition to these special features, attention is called to the following general features of the work:

1. The aim has been to make the work thoroughly inductive in its character. Ideas are presented before words, processes before rules,

analyses before inferences, and in every case the pupil is led to the ideas by easy, natural, and gradual steps. As a result of this method, all statements of principles, methods, definitions, etc. on the part of the pupil are merely the expression of what he already clearly understands. He is thus trained, not merely to follow old paths, but to be an independent truth-seeker.

2. The proper gradation of the work will be found a distinctive and valuable feature. Great pains have been taken to avoid those sudden transitions from the easy to the difficult which confuse and discourage pupils. The object has been to make each lesson an easy stepping-stone to the one which follows, so that pupils may pass from the simple to the complex with ease and delight.

3. The effort has also been to give a practical character to the work. It deals so far as possible with the objects most familiar to pupils. Its problems are designed to represent the actual things and events of life. Its language and methods are suggestive of business life, and a number of problems on historical events, biographical dates, dates of celebrated inventions, etc. add interest to the study and present valuable information to the pupil.

4. Lastly, it is believed that the work will be found to embody the educational spirit of the times. The work is designed not merely to teach Arithmetic, but to train the intelligence of the child. Its method of reasoning is that of analysis, by which each truth flows in simple sequence from a preceding one. From these processes of analysis methods are derived by the simple inferences of induction. Analysis and induction thus run like a golden thread through the entire work, brightening its processes and binding its parts together into logical unity. It is thus adapted to teach pupils to *think* as well as to *work problems*—to *develop mind* as well as the *power of computation*.

Appreciating the favor extended to my previous works, and cherishing the hope that this new book will aid teachers in their work and become a favorite of many of the boys and girls of the country, I intrust it, as the expression of my latest thought in teaching the elements of Arithmetic, to the decision of a kind and indulgent public.

EDWARD BROOKS.

PHILADELPHIA, PA., *May 10, 1895.*

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INTRODUCTION.

SUGGESTION FOR ORAL INSTRUCTION.

THE first lessons in arithmetic should be given orally by the teacher, in connection with the use of a text-book by the pupils. Every teacher should be qualified to give such an oral course, and we present a few suggestions which may be of use to guide young teachers in the work.

COURSE OF INSTRUCTION.

The course of instruction in the elements of arithmetic includes the following subjects:

1. Ideas and Names of Numbers.
2. The Method of Writing Numbers.
3. The Elementary Results.
4. The Fundamental Operations.
5. The Elements of Fractions.
6. The Elements of Denominate Numbers.

By the *Elementary Results* we mean the *sums* of the *addition table* with their corresponding *differences*, and the *products* of the *multiplication table* with their corresponding *quotients*. By the *Fundamental Operations* we mean the method of adding, subtracting, multiplying, and dividing larger numbers than those of the tables of addition and multiplication.

IDEAS AND NAMES OF NUMBERS.

IDEAS.—The child's first lessons in numbers should be given with *visible objects*. We may use *blocks*, *sticks*, *books*, *beads*, *beans*, *lines* and *circles* on the board, etc. The *arithmetical frame* is the most convenient for general purposes. To give ideas of *groups* of ten we can use *bunches of matches*, *toothpicks*, *little sticks*, or *circles with dots* in them.

NAMES.—The names of numbers are taught in connection with the ideas, and both are given in *counting*. In teaching these names

make the pupils first familiar with them as far as *ten*. Then lead them to conceive *ten* as a *group*, and instead of saying *eleven*, *twelve*, etc. at first, teach the child to say *one and ten*, *two and ten*, etc.; *two tens*, *two tens and one*, *two tens and two*, etc. Taught in this way to name numbers, pupils will have no difficulty in learning to *write* them. They can subsequently be permitted to use the words *eleven*, *twelve*, etc.

THE WRITING OF NUMBERS.

Teach first the meaning of the characters from 1 to 9, and drill the children in making them. Do not then pass immediately to writing *ten*, *eleven*, etc., but show that *four and ten* is expressed thus, 14; *three and ten* thus, 13; *two and ten* thus, 12; *one and ten* thus, 11; and in this way lead them to see the necessity of a new character expressing *nothing*, to show that the 1 to denote *ten* (10) is in the second place, and thus introduce the 0.

Taught in this way, children will have no difficulty in understanding the principle of *place-value* in our method of notation. Afterward pass gradually to larger numbers as the pupils are prepared for it.

ADDITION AND SUBTRACTION.

As soon as the pupil knows a few numbers and their names he should be led to *unite* and *separate* them. Instruction in these two processes should be given in accordance with the following principles:

1. *Addition and Subtraction should be taught with visible objects.* This principle is founded on the law of mental development, and is evident to every thoughtful teacher.

2. *Addition and Subtraction should be taught together in the first lessons.* This is evident, since the ideas are so intimately related that as soon as the child sees the *sum* of two numbers he has the elements of their *difference*, and can be led to immediately see it. Thus, as soon as he knows that 2 and 3 are 5 he can see that 5 diminished by 3 is 2, or 5 diminished by 2 is 3.

3. *The pupils must be required to commit the elementary sums and differences.* These are the *alphabet of arithmetic*, and must be fixed in the memory. The exercises should run as far as 9 and 9, or, in accordance with custom, as far as 12 and 12.

4. After the pupils know the sums to 9 plus 9, and the differences as far as 18 minus 9, they should be taught to add and subtract numbers from 1 to 9 to *larger numbers*, expressed in tens and units.

5. Following these exercises they should be taught to add and subtract numbers not exceeding 9 to numbers not exceeding 99, in which they must "carry" and "borrow."

MULTIPLICATION AND DIVISION.

After the pupil is familiar with a few of the elementary sums and differences, he can begin to learn the elements of Multiplication and Division. Instruction in these processes should be given in accordance with the following principles:

1. *Multiplication should be taught as concise Addition.* Thus, the pupil should be taught that 2 times 3 are 6, because 3 + 3 are 6. Multiplication will thus be conceived as a derivative of Addition.

2. *Division should be taught as reverse Multiplication.* Thus, it should be shown that 6 contains 3 two times, because two 3's are 6. In this way the quotients can be immediately derived from the products.

3. *Multiplication and Division should be taught together in the first lessons.* This is evident from the intimate relation of the ideas. Thus, as soon as the pupil sees that two 3's are 6, he is ready to see that 6 equals two 3's or contains 3 *two* times.

4. *Pupils should be taught to construct the multiplication table for themselves.* This they can readily do if they have been taught that Multiplication is concise Addition.

5. *Pupils should be required to commit the elementary products and quotients.* That is, they should commit the multiplication table and be able to derive the quotients directly from it.

6. In order to *commit the multiplication table,* pupils should *make* it, *write* it, *study* it, *recite* it, and *use* it.

THE FUNDAMENTAL OPERATIONS.

In connection with these exercises in learning the elementary sums, differences, products, and quotients, the pupils should be trained to apply these results to the Arabic method of notation—that is, to written arithmetic proper.

In Addition they should be taught "to carry" when the sum exceeds nine; in Subtraction, "to borrow" when a term of the subtrahend exceeds the term in the minuend; in Multiplication, to multiply by the different terms of the multiplier; and in Division, the methods of Short and Long Division.

Pupils must be trained until they become expert in these processes. For this instruction a text-book with examples will be found almost indispensable. The methods of teaching are given in this book under Addition, Subtraction, Multiplication, and Division.

In this instruction pupils may be taught the reason for the operation along with the operation, or, as some teachers think preferable with beginners, *we may teach the method first and the reason for it afterward.*

THE ELEMENTS OF FRACTIONS.

The Elements of Fractions should be introduced in connection with the previous subjects, and be carried along with them.

These elements should be presented *in the concrete* by the use of *objects*, *lines*, *squares*, and *circles* on the *board*, etc.

The methods of analysis with fractions present an excellent exercise for the training of the reasoning powers.

THE ELEMENTS OF DENOMINATE NUMBERS.

The Elements of Denominate Numbers should be introduced in connection with the instruction in the "elementary results." In this way pupils become familiar with the principal measures before they take up the subject formally in the book.

These elements should be presented *concretely* by having the different *units in the school-room*, when possible, for the pupils to see and handle. They should include the *foot*, *inch*, and *yard;* the *pint*, *quart*, and *gallon;* the *pound* and *ounce;* the *pint*, *quart*, and *peck* (dry measure); and the *day*, *hour*, *week*, and *month*.

Little problems in reducing from one unit to another can be given in connection with Multiplication and Division.

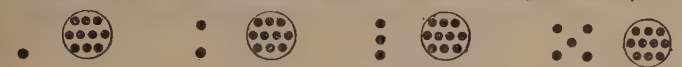
OTHER SUBJECTS IN ARITHMETIC.

Besides the subjects named as included in the elements of arithmetic, the present work embraces Factoring, Common Multiples and Divisors, Decimals, and Percentage. Suggestions for oral instruction in these subjects will be found in connection with the treatment of them.

NAMING NUMBERS.

1. In naming numbers pupils should be led to see clearly that when we have *ten* things we make a *group* of them and count them as *one of the group*.

2. Illustrate as below with *dots* representing units, and circles containing ten units representing tens, counting *one and ten, two and ten*, etc.

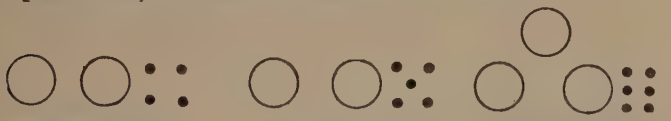


One and ten, Two and ten. Three and ten. Five and ten.

3. Illustrate also as below in counting *two tens and one, two tens and two*, etc.

Two tens and one. Two tens and two. Two tens and three.

4. Let the dots be omitted from the circle, and use the circle to represent tens, and count as follows:



Two tens and four. Two tens and five. Three tens and six.

5. Illustrate also that when we have ten of these *tens* we call it *hundred*. Represent the *hundred* by a larger circle, *tens* by a smaller circle, and *ones* by dots.

6. Have exercises in naming numbers represented by these characters, as follows:

One hundred, 1 ten, and 1. One hundred, 2 tens, and 3. Two hundred, 3 tens, and 4.

NOTE.—The ease or difficulty with which pupils learn to *write* numbers depends almost entirely on how they have been taught to *name* numbers. When pupils are taught to count by saying *one and ten, two and ten*, etc., *two tens and one, two tens and two*, etc., they will experience no difficulty in learning to write numbers.

WRITING NUMBERS.

1. In teaching pupils to write numbers, teach first the nine digits, *1, 2, 3, 4, 5, 6, 7, 8, 9*, and drill pupils until they can make them and know what they denote.

2. In teaching to write numbers beyond 9, begin with *fourteen*, or *four and ten*, and lead them to see that we write "4 and 1 ten" thus, *14*, the 4 denoting the *four* and the 1 denoting the 1 *ten*.

3. Next lead them to see that we write *five and ten* (fifteen), *three and ten* (thirteen), *two and ten* (twelve), *one and ten* (eleven), in a similar manner.

4. Then lead them to see that to write *one ten* we need a character to take the place of the 4, 3, 2, and 1 in 14, 13, 12, 11, to show that the 1 ten is in the *second place*. We might use a *square* first to denote no units, as in the margin, and, then rounding the corners, reach the *cipher*.

1□ One ten.

10 One ten.

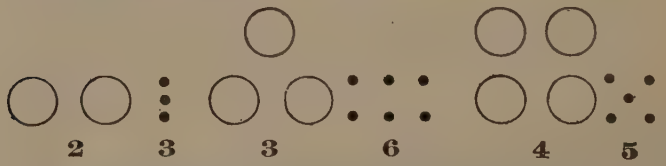
20 Two tens.

30 Three tens.

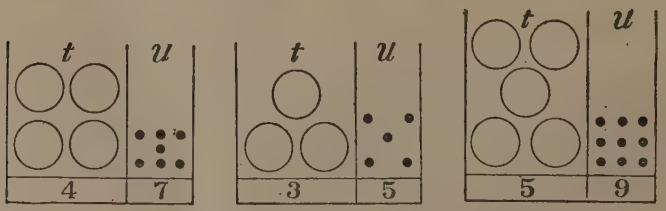
5. The pupil can then be easily led to see how to express 2 *tens*, 3 *tens*, etc., and will have no difficulty in expressing numbers as far as 99.

6. The method of expressing *one hundred* may be taught in a similar manner. Taught in this way, pupils will find very little difficulty with notation.

7. The subject may be illustrated with the *dots* and *circles* used in naming numbers. Thus:



8. We may also draw vertical lines and use these dots and circles to represent the number of *units*, *tens*, and *hundreds*. Thus:



SECTION I.

NUMBERS TO TEN.

LESSON I.

Numbers One and Two.

1. How many boys do you see in the picture?

2. How many girls do you see in the picture?

3. How many feet has the boy?

4. How many hands has the girl?

5. How many thumbs on your right hand?

6. How many thumbs on your left hand?

7. How many thumbs on both hands?

8. One thumb and one thumb are how many thumbs?

9. Make one short mark on your slate. |

10. Place one mark beside it, thus: | |

11. One mark and one mark are how many marks?

12. If we take one mark away, how many marks remain?

13. One and one are how many? One from two leaves how many?

14. Write the *word one* on your slate. *One.*

15. Write the *figure* for *one* on your slate. *1.*

16. Write the word *two* on your slate. *Two.*

17. Write the *figure* for two on your slate. *2.*

18. Make two rings, thus: ○ ○. ○ and ○ are how many rings?

19. If we take ○ from ○ ○, how many rings remain?

20. How many are two and one more? *Three.*

LESSON II.

The Number Three.

1. How many girls are there in the picture? How many boys?

2. How many children in the picture? Two and one are how many?

3. If one girl should go away, how many children would be left? One from 3 leaves how many?

4. If the 2 girls should go away, how many children would be left? Two from three leaves how many?

5. What do you see in the basket? If 1 of them jumps out, how many will remain?

6. If the boy takes 2 out, how many will be left? Two from three leaves how many?

7. Show with your blocks that 2 and 1 are three. Show that three "take away" 1 is 2.

8. Show that 1 and 2 are three. Show that three "take away" 2 is 1.

9. Make two marks on your slate. | |

10. Place one mark beside them, thus: | | |

11. Two marks and one mark are how many?

12. One mark from three marks leaves how many?

13. Tell a little number story about 2 birds and 1 bird.

14. Tell a little number story about three kittens.

15. Write the word *three* on your slate. *Three.*

16. Write the *figure* for three on your slate. *3.*

17. Make three triangles, thus: △ △ △. How many sides has each triangle?

18. How many triangles are △ △ and △?

19. How many are three and one more? *Four.*

LESSON III.

The Number Four.

1. How many feet has this horse?

2. How many wheels has the wagon?

3. How many fore feet has the horse? How many hind feet?

4. How many are two feet and two feet?

5. How many fore wheels has the wagon? How many hind wheels?

6. Two wheels and two wheels are how many wheels? Two and two are how many?

7. If I take one wheel off, how many wheels will remain? One wheel from four wheels leaves how many wheels?

8. If I take two wheels off, how many wheels will remain? Two wheels from four wheels leave how many wheels?

9. How many are 2 and 2? 3 and 1? 1 and 3?

10. How many are one from four? Two from four?

11. Write the word *four* on your slates. *Four.*

12. Write the *figure* that stands for four. *4.*

13. Suppose we use + for *and*, and = for *are*, then read, $2 + 2 = 4$; $3 + 1 = 4$; $1 + 3 = 4$.

14. Tell a little number story about $2 + 1$; about $3 + 1$; about $2 + 2$.

15. Copy and complete: $1 + 2 = ?$ $2 + 2 = ?$ $3 + 1 = ?$ $1 + 3 = ?$ $2 + 1 = ?$

16. Make a square thus: □. How many sides has it?

17. Take your blocks and tell me how many 2's make 4. How many 2's in 4?

18. How many are four and one more? *Five.*

LESSON IV.

The Number Five.

1. How many butterflies in the picture?

2. How many roses on the bush?

3. Three roses and two roses are how many roses?

4. Two roses and 3 roses are how many roses?

5. If 3 butterflies fly away, how many would remain?

6. If I should break off 2 roses, how many would remain?

7. How many are 3 and 2? 2 and 3? Four and one?

8. How many remain if 3 are taken from five? If 2 are taken from five? 4 from five?

9. How many roses must I take away to leave only 2?

10. How many butterflies must fly away to leave one?

11. How many are $4 + 1$? $3 + 2$? $2 + 2$? $2 + 3$?

12. The figure used for the word *five* is *5.*

13. Show with blocks how many are 4 "take away" 2? 5 "take away" 2? 5 "take away" 3?

14. If we use — for "take away," read, $4 - 1 = 3$; $5 - 2 = 3$; $5 - 3 = 2$; $5 - 4 = 1$.

15. Copy and complete: $2 + 1 = ?$ $4 - 2 = ?$ $3 + 2 = ?$ $5 - 2 = ?$ $5 - 3 = ?$

16. Tell a little story about 2 birds and 3 birds.

17. Tell a little story about 5 roses and 3 roses.

18. How many are five and one more? *Six.*

LESSON V.

The Number Six.

1. How many boys in the picture?

2. How many have flags? How many have guns?

3. How many are 2 boys and 2 boys?

4. How many are 3 boys and 3 boys?

5. If 2 boys leave, how many will remain?

6. How many are 2 boys taken from six boys? 2 from six are how many?

7. How many are $3 + 2$? $4 + 2$? $3 + 3$? $5 - 2$? $5 - 3$?

8. The figure in use for the word *six* is *6.*

9. Read, $4 + 2 = 6$; $6 - 2 = 4$; $3 + 3 = 6$; $6 - 3 = 3$; $6 - 4 = 2$.

10. Copy and complete: $4 + 2 = ?$ $5 - 3 = ?$ $3 + 3 = ?$ $6 - 3 = ?$ $6 - 2 = ?$ $6 - 4 = ?$

11. How many feet have the first 3 boys? How many are three 2's?

12. Look at the boys and tell me how many 3's make 6. How many 3's then in 6?

13. Take your blocks and show how many are two 2's; two 3's; three 2's.

14. With blocks show how many 2's in 4; 2's in 6; 3's in 6.

15. How many are two 2's? Three 2's? Two 3's? How many 2's in 4? 3's in 6? 2's in 6?

16. How many cents will three 2-cent postage-stamps cost?

17. How many 2-cent postage-stamps can you buy for 6 cents?

18. If you had 4 cents, how many apples could you buy at 2 cents each?

19. If you had 6 cents, how many apples could you buy at 3 cents each?

20. How many are six and one more? *Seven.*

LESSON VI.

The Number Seven.

1. How many boys in this picture? What are they riding?

2. How many are in front? How many behind?

3. How many boys are 4 boys and 3 boys?

4. If 3 boys ride out of sight, how many boys will remain? Three from seven are how many?

5. If 4 boys leave, how many boys remain? Four from seven are how many?

6. Three and four are how many? Two from seven leaves how many?

7. Show with blocks how many are 4 and 3; 5 and 2; 6 and 1; 3 and 4.

8. Show with blocks how many are seven take away 4; seven take away 3; seven take away 2.

9. How many days in a week? Name the days. How many days from Sunday to Thursday?

10. If a boy goes to school 5 days in a week, how many days of the week is he at home?

11. We write for the word *seven* the figure 7.

12. Read, $3 + 2 = 5$; $4 + 3 = 7$; $5 + 2 = 7$; $3 + 4 = 7$; $2 + 5 = 7$.

13. Read, $7 - 2 = 5$; $7 - 3 = 4$; $7 - 4 = 3$; $7 - 5 = 2$; $7 - 6 = 1$.

14. If you had 5 apples and bought 2 more, how many apples would you then have?

15. If you have 7 cents and give 4 cents away, how many cents remain?

16. If Elsie had 7 oranges and gave Alice 3 of them, how many would Elsie still have?

17. Show with blocks how many 2's in 4; 2's in 6; 3's in 6.

18. Copy and complete: $3 + 2 = ?$ $4 + 2 = ?$ $5 + 2 = ?$ $4 + 3 = ?$ $2 + 5 = ?$

19. Copy and complete: $4 - 2 = ?$ $6 - 4 = ?$ $7 - 3 = ?$ $7 - 2 = ?$ $7 - 5 = ?$ $7 - 6 = ?$

20. Tell little number stories about the following numbers: $3 + 2$; $6 - 4$; $5 + 2$; $4 + 3$; $6 - 2$.

21. Find the results of the following, and write them under the lines;

4	5	6	5	4	7	7	7
$+\underline{2}$	$+\underline{2}$	$-\underline{4}$	$-\underline{2}$	$+\underline{3}$	$-\underline{2}$	$-\underline{3}$	$-\underline{5}$

22. How many are seven and one more? *Eight.*

LESSON VII.

The Number Eight.

1. How many children in the class?

2. How many boys in the class? How many girls?

3. How many pupils are 5 boys and 3 girls?

4. If the 3 girls should leave, how many pupils would remain? Three from eight leaves how many?

5. If the 5 boys should leave, how many pupils would remain? Five from eight leaves how many?

6. How many are 4 and 4? 5 and 3? 6 and 2? 3 and 5? 2 and 6?

7. How many blocks on the right side of the table? How many blocks in each row?

8. How many blocks are 4 blocks and 4 blocks? How many are two 4's?

9. We write for the word *eight* the figure *8.*

10. How many are 2 from 8? 3 from 8? 4 from 8? 5 from 8? 6 from 8?

11. Read, $5 + 3 = 8$; $4 + 4 = 8$; $6 + 2 = 8$; $7 + 1 = 8$; $3 + 5 = 8$.

12. Read, $8 - 2 = 6$; $8 - 3 = 5$; $8 - 4 = 4$; $8 - 5 = 3$; $8 - 6 = 2$.

13. Show each of the results of the two previous examples with your blocks.

14. In the picture, see how many 4's make 8. How many 4's are in 8?

15. In the picture, see how many 2's make 8. How many 2's in 8?

16. How many are two 2's? Three 2's? Four 2's? Two 3's? Two 4's?

17. Tell little number stories about the figures in the two examples at the top of the page.

18. How many cents must you pay for three 2-cent postage-stamps?

19. How many cents must you pay for four 2-cent postage-stamps?

20. If Willie had 8 little rabbits in a box and 3 jumped out, how many would remain in the box?

21. If you have 8 cents, how many oranges can you buy at 4 cents apiece?

22. If you had 8 cents, how many apples could you buy at 2 cents each?

23. How many 2-cent postage stamps can you buy for eight cents?

24. Copy and complete: $5 + 2 = ?$ $6 + 2 = ?$ $7 + 1 = ?$ $5 + 3 = ?$ $4 + 4 = ?$

25. Copy and complete: $8 - 2 = ?$ $8 - 4 = ?$ $8 - 3 = ?$ $8 - 5 = ?$ $8 - 6 = ?$ $8 - 7 = ?$

26. Find the results of the following:

5	6	4	3	6	7	3	2
+2	+1	+4	+5	+2	+1	+5	+6

27. How many are 8 and 1 more?

LESSON VIII.

The Number Nine.

1. How many birds do you see in the picture?

2. How many birds on the ground? How many on the fence? How many in the tree?

3. How many are 3 and 3 and 3? How many are three 3's?

4. If the three birds on the tree should fly away, how many would be left?

5. Three from nine leaves how many?

6. How many birds on the ground and the fence? If these 6 should fly away, how many would be left?

7. Six from nine leaves how many?

8. How many apples on the right side of the tree? How many on the left side? How many in all?

9. How many apples are 5 apples and 4 apples?

10. If four apples should fall off the tree, how many would remain on? Four from nine leaves how many?

11. Show with blocks how many are three 3's. Show how many 3's make nine. How many 3's in nine?

12. How many are 3 and 6? 6 and 3? 5 and 4?

13. We write for the word *nine* the figure *9.*

14. Read, $6 + 3 = 9$; $7 + 2 = 9$; $5 + 4 = 9$; $4 + 5 = 9$; $9 - 2 = 7$; $9 - 4 = 5$; $9 - 3 = 6$.

15. Tell little number stories of each of the above examples.

16. How many are two 2's? Three 2's? Four 2's? Two 3's? Two 4's? Three 3's?

17. How many 2's in 4? 2's in 6? 3's in 6? 2's in 8? 4's in 8? 3's in 9?

18. Copy and complete: $7 + 2 = ?$ $5 + 4 = ?$ $9 - 2 = ?$ $9 - 3 = ?$ $9 - 4 = ?$

19. Write the results of the following:

5	6	7	9	9	9	9	9
+4	+3	+2	−2	−3	−5	−4	−6

20. How many are one more than nine? *Ten.*

LESSON IX.

The Number Ten.

1. How many chickens do you see in the picture?

2. How many on the right side? How many on the left side?

3. How many are 5 and 5? How many are two 5's?

4. How many black chickens do you see? How many white chickens?

5. How many are 6 and 4? How many are 4 and 6?

6. If the white chickens should run away, how many would be left? How many are 4 from ten?

7. Tell me a story about the little girl and the chickens.

8. How many are 8 and 2? 7 and 3? 6 and 4? 5 and 5? 4 and 6?

9. How many are ten "take away" 2? Ten "take away" 4? Ten "take away" 5? Ten "take away" 3? Ten "take away" 6?

10. We write for the word *ten* the characters *10.*

11. Read, $5 + 5 = 10$; $6 + 4 = 10$; $7 + 3 = 10$; $8 + 2 = 10$.

12. Read, $10 - 2 = 8$; $10 - 3 = 7$; $10 - 4 = 6$; $10 - 5 = 5$.

13. Tell little number stories about each of the above examples.

14. How many 2's can you count in the chickens in the picture? How many 2's make 10?

15. How many 5's can you count in the picture? How many 5's make 10?

16. How many are two 2's? Three 2's? Four 2's? Five 2's? Two 3's? Two 4's? Two 5's?

17. How many 2's make 10? How many 2's in 10? How many 5's in 10?

18. If you have 10 cents and spend 2 cents for candy, how many cents will remain?

19. How many 2-cent stamps can you buy for 10 cents?

20. How many 5-cent stamps can you buy for 10 cents?

21. Edith bought three 2-cent stamps and gave the clerk a ten-cent piece; what change should she receive?

22. Copy and complete: $8 + 2 = ?$ $7 + 3 = ?$ $6 + 4 = ?$ $10 - 2 = ?$ $10 - 3 = ?$ $10 - 5 = ?$

23. Write the results of the following:

$$\begin{array}{rrrrrrrrrr} 6 & 7 & 8 & 9 & 10 & 10 & 10 & 10 & 10 \\ +\underline{4} & +\underline{3} & +\underline{2} & +\underline{1} & -\underline{2} & -\underline{3} & -\underline{4} & -\underline{5} & -\underline{6} \end{array}$$

LESSON X.

Review Adding and Subtracting up to Ten.

1. How many are 2 and 2? 4 and 2? 3 and 2? 5 and 2? 7 and 2? 6 and 2? 8 and 2?

2. How many are 2 from 4? 2 from 6? 2 from 5? 2 from 7? 2 from 8? 2 from 9? 2 from 10?

3. How many are 2 and 3? 4 and 3? 3 and 3? 5 and 3? 6 and 3? 7 and 3?

4. How many are 3 from 4? 3 from 6? 3 from 5? 3 from 7? 3 from 9? 3 from 8? 3 from 10?

5. How many are 2 and 4? 4 and 4? 3 and 4? 5 and 4? 6 and 4?

6. How many are 4 from 5? 4 from 6? 4 from 8? 4 from 7? 4 from 9? 4 from 10?

7. How many are 1 and 5? 2 and 5? 4 and 5? 3 and 5? 5 and 5?

8. How many are 5 from 6? 5 from 7? 5 from 9? 5 from 8? 5 from 10?

9. How many are 1 and 6? 3 and 6? 2 and 6? 4 and 6? 3 and 7? 1 and 8? 2 and 8? 1 and 9?

10. How many wheels has a wagon? How many has a cart? How many wheels have both?

11. If a boy had 4 marbles in one pocket and 3 in another, how many had he in both pockets?

12. How many cents will it take to buy a 5-cent kite and a 3-cent ball?

13. If Mabel had 6 roses and gave her sister 4 of them, how many would she still have?

14. If a boy lost 4 cents and had 4 cents left, how many cents had he at first?

15. If Ethel had 8 apples and gave 2 of them to Annie and 2 to Lizzie, how many apples would Ethel still have?

LESSON XI.

Review Multiplying up to Ten.

1. If I pick up 2 blocks with one hand and 2 blocks with the other hand, how many times 2 blocks do I pick up?

2. How many blocks are 2 times 2 blocks? How many blocks are 3 times 2 blocks?

3. Show with your blocks how many are 4 times 2 blocks; 5 times 2 blocks.

4. A short way of writing "2 times 2 equals 4" is $2 \times 2 = 4$. The sign $\times$ means *times*.

5. Read, $2 \times 2 = 4$; $2 \times 3 = 6$; $2 \times 4 = 8$; $2 \times 5 = 10$; $3 \times 2 = 6$; $4 \times 2 = 8$.

6. How many are two 3's? 2 times 3? Three 3's? 3 times 3? Four 2's? 4 times 2?

7. How many are two 4's? 2 times 4? Two 5's? 2 times 5? Five 2's? 5 times 2?

8. Write with the sign $\times$ the following: 2 times $2 = 4$; 3 times $2 = 6$; 4 times $2 = 8$; 5 times $2 = 10$.

9. Write with the sign $\times$ the following: 2 times $3 = 6$; 3 times $3 = 9$; 2 times $4 = 8$; 2 times $5 = 10$.

10. Copy and complete: $2 \times 2 = ?$ $2 \times 3 = ?$ $2 \times 4 = ?$ $2 \times 5 = ?$ $3 \times 2 = ?$ $3 \times 3 = ?$ $4 \times 2 = ?$

11. If I give two little girls each 3 cents, how many cents will I give away?

12. A lady gave 4 cents to each of two little boys; how many cents did she give away?

13. Four little girls each gave 2 cents to a poor woman; how many cents did the woman receive?

14. Write the results of the following:

2	2	2	2	3	3	4	5
$\times 2$	$\times 3$	$\times 4$	$\times 5$	$\times 2$	$\times 3$	$\times 2$	$\times 2$

LESSON XII.

Review Dividing up to Ten.

1. Show with blocks or sticks how many 2's in 4; how many 2's in 6; how many 2's in 8.

2. Tell me how many 2's in 4; how many 2's in 6; how many 2's in 8.

3. Show by blocks or sticks how many 3's in 6; how many 3's in 9; how many 4's in 8.

4. Tell me how many 3's in 6; how many 3's in 9; how many 4's in 8; how many 5's in 10.

5. If I have 6 apples, to how many little boys can I give 2 apples each?

6. With blocks or sticks divide 4 into 2's; 6 into 3's; 8 into 4's; 9 into 3's; 10 into 5's.

7. Since there are 2 pints in 1 quart, how many pints of water will a quart measure contain?

8. Since there are three 2's in 6, how many 2's does 6 contain?

9. Since there are four 2's in 8, how many 2's does 8 contain? How many fours does 8 contain?

10. We write 6 divided into 2's, thus: $6 \div 2$. We write 8 divided into 2's thus: $8 \div 2$.

11. If we write $6 \div 3 = 2$, the 2 expresses how many 3's in 6. It means there are two 3's in 6, or the 3's in 6 are 2.

12. Explain the following: $4 \div 2 = 2$; $8 \div 2 = 4$; $8 \div 4 = 2$; $6 \div 2 = 3$; $10 \div 2 = 5$; $10 \div 5 = 2$.

13. We may also read the expression $6 \div 2 = 3$, "six divided by 2 equals 3."

14. Read the following: $4 \div 2 = 2$; $8 \div 2 = 4$; $6 \div 3 = 2$; $8 \div 4 = 2$; $10 \div 5 = 2$.

15. Copy and complete the following: $4 \div 2 = ?$ $6 \div 2 = ?$ $6 \div 3 = ?$ $8 \div 2 = ?$ $8 \div 4 = ?$ $9 \div 3 = ?$ $10 \div 2 = ?$ $10 \div 5 = ?$

LESSON XIII.

One-half and One-fourth.

1. If we divide an apple into two equal parts, what is one part called? *Ans. One-half.*

2. Make a circle and divide it into two equal parts. What is one part called?

3. If we divide four blocks into two equal parts, how many blocks in one of the equal parts?

4. What is one of the two equal parts of 4? What is one-half of 4?

5. What is one of the two equal parts of 6? What is one-half of 6?

6. What is one-half of 4? One-half of 6? One-half of 8? One-half of 10?

7. If we divide a square into four equal parts, what is one part called? *Ans. One-fourth.*

8. Make a circle and divide it into four equal parts. What is one part called? What shall we call two parts?

9. Divide 8 blocks or sticks into 4 equal parts. How many in each part? What is one-fourth of 8?

10. Divide 2 into two equal parts. How many in each part? What is one-half of 2?

11. Divide 4 into four equal parts. How many in each part? What is one-fourth of 4?

12. If I had 4 little birds and gave one-half of them to Susan, how many birds did I give to Susan?

13. If I had 8 oranges and gave one-half of them to Robert, how many oranges did I give to Robert?

14. A little girl had 10 cents, and spent one-half of them for candy; how many cents did she spend?

LESSON XIV.

The Cent, Nickel, and Dime.

1. Let us look at a cent. Tell me some things about it. How do the two sides differ?

2. Let us look at a 5-cent piece. Tell me some things about it. How do the two sides differ? What other name has it?

3. How many cents make a nickel? How many cents in 2 nickels?

4. If you had a nickel and spent 3 cents, how many cents would remain?

5. What change would you receive if you gave a clerk a nickel for 2 two-cent stamps?

6. Let us look at a dime. Tell me some things about it.

7. How many cents in a dime? How many nickels in a dime?

8. What part of a dime is a nickel? How many cents in half a dime?

9. A nickel and how many cents make a dime? A nickel is how many cents less than a dime?

10. At 2 cents each what will 5 postage-stamps cost?

11. How many 5-cent postage-stamps can you buy for a dime?

12. How many 2-cent postage stamps can you buy for a dime?

13. Susan's mother gave her a nickel and 3 cents; how many more cents will make a dime?

14. How many cents will 2 car-fares cost at a nickel each?

15. How many 5-cent car-fares can you pay for with a dime?

16. What two pieces of money will make 6 cents? What two pieces of money will make 10 cents?

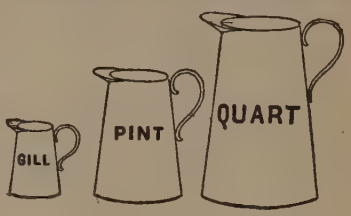
17. What three pieces of money will make 7 cents?

LESSON XV.

The Quart, Pint, and Gill.

1. Look at these measures.

2. Name the measures. Which is the smallest? Which is the largest?



3. Let us fill the pint measure with sand. Pour the sand into the quart measure. Do this until the quart measure is full.

4. How many pints of sand will fill the quart measure?

5. How many pints of water will make a quart of water? How many pints of milk will make a quart of milk?

6. If I have a quart of berries, how many pints have I?

7. Mary has 2 quarts of chestnuts; how many pints has she?

8. Four pints of water will fill how many quart measures?

9. Eight pints of milk will fill how many quart cans?

10. If a pint of milk costs 3 cents, what will a quart of milk cost? What will 2 quarts cost?

11. Fill the gill measure with sand; empty it into the pint measure. Do this until the pint measure is full.

12. How many gills of sand will fill the pint measure? How many gills of water will fill the pint measure?

13. How many gills make a pint? How many gills make 2 pints?

14. Edith bought a pint of milk; how many gills of milk did she buy?

15. How many pints make a quart? How many gills make a pint?

16. How many gills make 2 pints or one quart? How many gills in a quart?

17. What part of a quart is one pint? What part of a pint is one gill?

18. In 2 pints of milk how many gills? In 2 quarts of milk how many pints?

19. Look at this tin cup. Does it contain nearly a pint or nearly a gill?

20. Here is a common-sized tumbler. How many gills does it hold? What part of a pint does it contain?

REVIEW EXERCISES.

$3 + 2 = ?$	$4 - 2 = ?$	$2 \times 2 = ?$	$2 \div 2 = ?$
$2 + 4 = ?$	$6 - 2 = ?$	$2 \times 3 = ?$	$4 \div 2 = ?$
$5 + 3 = ?$	$5 - 3 = ?$	$3 \times 2 = ?$	$6 \div 2 = ?$
$6 + 2 = ?$	$7 - 2 = ?$	$2 \times 4 = ?$	$5 \div 1 = ?$
$4 + 5 = ?$	$6 - 4 = ?$	$3 \times 3 = ?$	$6 \div 3 = ?$
$5 + 2 = ?$	$3 - 2 = ?$	$4 \times 2 = ?$	$8 \div 2 = ?$
$1 + 6 = ?$	$5 - 2 = ?$	$7 \times 3 = ?$	$4 \div 4 = ?$
$2 + 7 = ?$	$6 - 4 = ?$	$5 \times 2 = ?$	$6 \div 1 = ?$
$5 + 4 = ?$	$7 - 5 = ?$	$6 \times 1 = ?$	$8 \div 4 = ?$
$8 + 2 = ?$	$8 - 4 = ?$	$1 \times 8 = ?$	$6 \div 6 = ?$
$7 + 3 = ?$	$6 - 1 = ?$	$2 \times 5 = ?$	$9 \div 3 = ?$
$6 + 4 = ?$	$9 - 4 = ?$	$4 \times 3 = ?$	$10 \div 2 = ?$
$2 + 8 = ?$	$10 - 6 = ?$	$8 \times 1 = ?$	$10 \div 5 = ?$

RAPID WORK.

$6 - 4 + 5 - 3 = ?$	$8 + 2 - 6 + 4 = ?$
$5 + 3 - 4 + 2 = ?$	$2 + 7 - 5 + 3 = ?$
$4 + 6 - 5 + 3 = ?$	$9 - 5 + 6 - 4 = ?$
$7 - 4 + 6 - 4 = ?$	$6 + 4 - 5 + 2 = ?$
$3 + 6 - 7 + 5 = ?$	$8 - 5 + 4 - 5 = ?$

SECTION II.

NUMBERS FROM TEN TO TWENTY.

LESSON I.

1. Take ten sticks and put a string around them.

2. Make ten dots on your slate and draw a circle around them.

3. One more than ten is how many?

Ans. One and ten, called *Eleven*, written 11.

4. Two more than ten are how many?

Ans. Two and ten, called *Twelve*, written 12.

5. Three more than ten are *three* and *ten*, called *Thirteen*, written 13.

6. Four and ten are called *Fourteen*, written 14.

7. Five and ten are called *Fifteen*, written 15.

8. Six and ten are called *Sixteen*, written 16.

9. Seven and ten are called *Seventeen*, written 17.

10. Eight and ten are called *Eighteen*, written 18.

11. Nine and ten are called *Nineteen*, written 19.

12. If I have two bunches of sticks, each containing ten, how many tens have I?

Ans. Two tens, called *Twenty*, written 20.

13. In 14 what does the 4 denote? What does the 1 denote? In 13 what does the 3 denote? The 1 denote?

14. The characters 1, 2, 3, 4, etc. are called *figures*.

15. What does each of the figures denote in 15? In 16? In 17? In 18? In 19?

16. In 10 the 0, or *cipher*, denotes no *ones* or *units*.

17. In 10 what does the 1 denote? *Ans.* One *ten*.

LESSON II.

The Number Eleven.

1. Take eleven sticks or blocks. Separate them into 10 and 1. Into 9 and 2. Into 8 and 3.

2. Show with sticks how many are 9 and 2; 2 from 11; 9 from 11.

3. If I had 9 roses, and found 2 more, how many roses would I have?

4. Mary had 11 roses, and lost two of them; how many remained?

5. Show with sticks how many are 8 and 3; 3 from 11; 8 from 11.

6. There are 8 boys and 3 girls in a class; how many children in the class?

7. If there were 11 eggs in a nest, and 3 were taken out, how many were left?

8. Tell with sticks, and then without them, how many are 7 and 4; 4 from 11; 4 and 7; 7 from 11.

9. Elsie had 11 cents and spent 4 cents; how many cents had she left?

10. How many are 6 and 5? 5 and 6? 5 from 11? 6 from 11?

11. How many marbles must I put with 6 marbles to have 11 marbles?

12. Copy and complete: $10 + 1 = ?$ $9 + 2 = ?$ $8 + 3 = ?$ $7 + 4 = ?$ $6 + 5 = ?$ $11 - 1 = ?$ $2 + 9 = ?$ $3 + 8 = ?$ $4 + 7 = ?$ $5 + 6 = ?$

Add:										
	3	2	4	3	7	4	2	3	4	4
	4	5	1	2	1	2	6	5	1	5
	2	4	6	5	3	3	3	2	4	2

Subtract:										
	11	11	11	11	11	11	11	11	11	11
	2	4	6	3	5	7	9	8	1	10

LESSON III.

The Number Twelve.

1. Take twelve sticks or blocks. Separate them into 10 and 2. Into 9 and 3. Into 8 and 4. Into 7 and 5. Into 6 and 6.

2. Show with sticks how many are 10 and 2; 2 and 10; 2 from 12; 10 from 12.

3. Show with sticks how many are 9 and 3; 3 and 9; 3 from 12; 9 from 12.

4. Tell me without using sticks how many are 9 and 3; 3 and 9; 3 from 12; 9 from 12.

5. Tell me, first with sticks, then without them, how many are 8 and 4; 4 and 8; 4 from 12; 8 from 12.

6. How many are 7 and 5? 5 and 7? 5 from 12? 7 from 12? 6 and 6? 6 from 12?

7. Bertha has 10 chestnuts, and Bessie gives her two more; how many chestnuts will Bertha then have?

8. There were 12 birds on a fence, and 4 of them flew away; how many remained?

9. How many are six 2's? How many are two 6's?

10. How many 2's in 12? How many 6's in 12?

11. How many are four 3's? Three 4's? How many 3's in 12? 4's in 12?

12. If one apple costs 2 cents, what will 6 apples cost?

13. If one orange costs 4 cents, what will 3 oranges cost?

14. Draw 4 triangles. How many sides have they all?

15. Draw 3 squares. How many sides have they all?

16. How many 2-cent postage-stamps can I buy for 12 cents?

17. How many 4-cent oranges can I get for 12 cents?

18. What is one-half of 12? One-fourth of 12? One-third of 12?

LESSON IV.

Foot, Yard, and Dozen.

1. Look at this rule. It is one foot long.

2. Look at the rule, and then mark the length of a foot on the board.

3. Measure the length of a foot on the board with the rule.

4. Measure the length of your desks. How many feet long are they?

5. Measure the height of your desks. How many feet high are they?

6. Notice the lines that run all the way across the rule.

7. Into how many parts do they divide the rule? Each part is called an *inch.*

8. How many inches in a foot-rule? How many inches make a foot?

9. How many inches wide is your slate? How many inches long is your reader?

10. How many inches in one-half of a foot? How many inches in one-fourth of a foot?

11. How long is your thumb to the end of the first joint?

12. See this longer rule. It is called a *yard.*

13. Let us measure the length of the blackboard with the yard. How many yards long is it?

14. Notice that the yard is divided into three equal parts. How long is each of these parts?

15. How many feet long is a yard? How many feet make a yard?

16. Twelve things of any kind are called a *dozen.*

17. Show me a dozen sticks. A dozen blocks. A dozen marks on your slate.

18. Alice had a dozen apples, and gave me 5 of them; how many had she left?

19. Willis had a dozen oranges, and gave one-half of them to Wallis; how many did he give away?

20. Bertie had a dozen marbles, and lost one-fourth of them; how many did he lose? How many did he have left?

21. At 4 cents a dozen how many marbles can I buy for 1 cent? How many marbles can I buy for half a dime?

22. Take a foot-rule. Mark its length with a line on the board. How many inches in a foot-rule? Mark the length of an inch on the board.

23. How many pins in a dozen? How many pins in one-half a dozen?

24. Copy and complete: $10 + 2 = ?$ $9 + 3 = ?$ $8 + 4 = ?$ $7 + 5 = ?$ $6 + 6 = ?$ $12 - 2 = ?$ $12 - 3 = ?$ $12 - 4 = ?$ $12 - 5 = ?$ $12 - 6 = ?$

25. Copy and complete: $2 \times 6 = ?$ $12 \div 2 = ?$ $4 \times 3 = ?$ $12 \div 3 = ?$ $1 \times 12 = ?$ $6 \times 2 = ?$ $12 \div 6 = ?$ $3 \times 4 = ?$ $12 \div 4 = ?$ $12 \div 1 = ?$

Add:

6	2	3	4	5	6	7	3	9	1	1
2	5	7	3	6	3	2	1	1	1	0
4	5	2	5	1	3	3	8	2	10	11

Subtract:

12	12	12	12	12	12	12	12	12	12	12
2	4	6	5	3	7	9	8	10	11	1

Multiply:

6	2	4	3	5	2	4	2
2	6	3	4	2	5	2	4

Divide:

$2)\overline{6}$	$2)\overline{8}$	$2)\overline{10}$	$2)\overline{12}$	$6)\overline{12}$
$4)\overline{8}$	$3)\overline{9}$	$5)\overline{10}$	$3)\overline{12}$	$4)\overline{12}$

LESSON V.

One-third and One-sixth.

1. Into how many equal parts have I cut this apple?

2. Each of these parts is called a *third.*

3. What are 2 of the parts called? How many thirds are there in an apple?

4. Into how many equal parts is this circle divided? What is each part called? What are two parts called?

5. How many thirds in the entire circle? How many thirds in anything?

6. Divide 6 blocks into three equal parts. How many blocks in each of these parts? What is one-third of 6?

7. Divide 9 blocks into 3 equal parts. What is one-third of 9?

8. Divide 12 blocks into 3 equal parts. What is one-third of 12?

9. Divide the square into two equal parts by drawing a line from left to right. What is each part called?

10. Now draw two lines from top to bottom, as shown in the margin.

11. Into how many equal parts is the square divided? What is each part called? What, then, is one-sixth?

12. Divide a circle into six equal parts. Divide a line into six equal parts.

13. Take 6 blocks and separate them into 6 equal parts. What is one-sixth of 6? One-sixth of 12?

14. Lucy had a dozen chestnuts, and ate one-third of them; how many did she eat? How many remained?

LESSON VI.

The Number Thirteen.

1. Take 13 objects. Separate them into 10 and 3. Into 11 and 2. Into 9 and 4. Into 8 and 5.

2. Show with counters how many are 9 and 4; 4 and 9; 4 from 13; 9 from 13.

3. Tell without counters how many are 9 and 4; 4 and 9; 4 from 13; 9 from 13.

4. Tell first with sticks, and then without them, how many are 8 and 5; 5 and 8; 5 from 13; 8 from 13.

5. How much must I pay for five 2-cent stamps and one 3-cent stamp?

6. How many are 7 and 6? 6 and 7? 6 from 13? 7 from 13?

7. If Mary has 9 pinks in one bunch and 4 in another, how many pinks has she in both bunches?

8. John had 13 cents, and spent 5 of them for postage-stamps; how many cents had he left?

9. Seven of my rosebushes are in blossom, and the other six are not; how many rosebushes have I?

10. How many are 7 and 6? 6 and 7? 6 from 13? 7 from 13?

11. How many are 8 and 5? 5 and 8? 5 from 13? 8 from 13? 9 and 4? 4 and 9? 4 from 13? 9 from 13?

12. Copy and complete: $9 + 4 = ?$ $8 + 5 = ?$ $7 + 6 = ?$ $13 - 9 = ?$ $6 + 7 = ?$ $13 - 4 = ?$

13. Copy and add:

2	4	3	5	2	1	4	5	4	6
5	2	4	2	8	9	2	4	5	2
5	7	4	6	3	2	3	3	4	5

14. Copy and subtract:

13	13	13	13	13	13	13	13	13	13
2	4	3	5	7	8	6	9	10	12

LESSON VII.

The Number Fourteen.

1. Take 14 sticks. Separate them into 10 and 4. Into 11 and 3. Into 9 and 5.

2. Show with sticks how many are 9 and 5; 8 and 6; 7 and 7; 10 and 4.

3. Tell without sticks how many are 10 and 4; 4 and 10; 4 from 14; 10 from 14.

4. How many are 7 and 7? How many are two 7's? How many are seven 2's?

5. How many days in a week? How many days in two weeks?

6. How much must I pay for seven 2-cent postage-stamps?

7. How many 7's in 14? How many 2's in 14?

8. How many 2-cent postage-stamps can I buy for 14 cents?

9. If one top costs 7 cents, how much will 2 tops cost?

10. How many pencils at 2 cents each can I get for 14 cents?

11. Mary was away from home 14 days; how many weeks was she away?

12. Mary has 9 white hens and 5 black hens; how many hens has she?

13. Jane had 14 ducks, and 6 of them flew away; how many ducks remained?

14. What is one-half of 14? If two loaves of bread cost 14 cents, what does one loaf cost?

15. How many are 11 and 3? 3 and 11? 3 from 14? 11 from 14?

16. How many are 12 and 2? 2 and 12? 2 from 14? 12 from 14? 9 and 5? 9 from 14?

17. How many are 5 and 9? 5 from 14? 8 and 6? 6 and 8? 6 from 14? 8 from 14?

18. Copy and complete:

$9 + 5 = ?$	$7 \times 2 = ?$	$14 - 6 = ?$	$12 + 2 = ?$
$8 + 6 = ?$	$14 - 7 = ?$	$14 - 8 = ?$	$14 - 9 = ?$
$2 \times 7 = ?$	$14 - 5 = ?$	$14 \div 2 = ?$	$14 \div 7 = ?$

19. Copy and add:

1	5	2	3	4	1	7	5	6	2
6	3	6	6	2	6	1	4	1	4
4	6	4	5	6	4	6	3	5	8

20. Copy and subtract:

14	14	14	14	14	14	14	14	14	14
2	5	3	6	4	7	1	8	10	12

LESSON VIII.

The Number Fifteen.

1. Take fifteen sticks. Separate them into 10 and 5. Into 13 and 2. Into 12 and 3.

2. With sticks show how many are 12 and 3; 9 and 6; 8 and 7; 6 and 9.

3. How many are 10 and 5? 5 from 15? 5 and 10? 10 from 15? 6 and 9? 9 from 15?

4. A boy had 11 marbles and found 4 marbles; how many marbles had he then?

5. How many are 9 and 6? 6 from 15? 8 and 7? 7 from 15? 8 from 15?

6. A girl had 15 little chickens, and 6 of them died; how many had she then?

7. Edith has two 5-cent pieces; how many more cents will it take to make 15 cents?

8. How many are 11 and 4? 4 from 15? 4 and 11? 11 from 15?

9. What two pieces of money make 15 cents? What three pieces of money?

10. How many are five 3's? How many are three 5's? How many are 5 times 3?

11. How much must I give for five 3-cent postage-stamps?

12. How many 3's in 15? How many 5's in 15? How many times is 5 contained in 15?

13. How many 3-cent postage-stamps can I buy for 15 cents?

14. There are 3 feet in one yard; how many feet are there in 5 yards?

15. A nickel is worth 5 cents; how many cents are 3 nickels worth?

16. How many yards are there in 15 feet? How many nickels in 15 cents?

17. Three times what number equals 15? Five times what number equals 15?

18. Make 5 triangles on your slate. How many sides are there in 5 triangles?

19. How many little sticks will it take to make 4 triangles?

20. How many triangles can you make out of 15 little sticks?

21. Copy and complete:

$9 + 6 = ?$ $\quad 5 \times 3 = ?$ $\quad 3 + 12 = ?$ $\quad 10 + 5 = ?$

$7 + 8 = ?$ $\quad 15 - 7 = ?$ $\quad 15 - 9 = ?$ $\quad 15 - 12 = ?$

$3 \times 5 = ?$ $\quad 15 - 5 = ?$ $\quad 15 \div 3 = ?$ $\quad 15 \div 5 = ?$

22. Copy and add:

5	5	7	5	6	2	4	3	6	6
3	4	4	2	5	7	9	3	4	7
7	6	4	8	4	6	2	9	5	2

23. Copy and subtract:

15	15	15	15	15	15	15	15	15	15
5	4	6	8	7	10	9	12	11	13

LESSON IX.

The Number Sixteen.

1. Take 16 sticks. Separate them into 10 and 6. Into 9 and 7. Into 8 and 8.

2. Show that 16 equals 10 and 6; 9 and 7; 8 and 8; 11 and 5; 12 and 4.

3. How many are 10 and 6? 6 from 16? 6 and 10? 10 from 16?

4. How many are 9 and 7? 7 from 16? 7 and 9? 9 from 16? 8 and 8? 8 from 16?

5. I saw 11 sparrows in a tree and 5 on the fence; how many sparrows did I see?

6. There were 16 sparrows on a tree, and 5 flew away; how many remained?

7. There were 9 ducks in the water and 7 on the ground; how many ducks in all?

8. There were 16 ducks in the water, and 8 of them came up on the bank; how many remained in the water?

9. How many are eight 2's? How many are four 4's? How many are two 8's?

10. How many 2's are there in 16? How many 4's in 16? How many 8's in 16?

11. Draw four squares. How many sides have the four squares?

12. There are 2 pints in one quart; how many quarts in 16 pints?

13. Separate 16 into 2 equal parts. What is one-half of 16?

14. If 2 tops cost 16 cents, how much will one top cost?

15. Separate 16 into 4's. What is one-fourth of 16?

16. Florence bought 4 paper dolls for 16 cents; what did each cost?

17. How many shoes will it take to shoe 4 horses?

18. If there are 4 quarts in one gallon, how many quarts in 3 gallons?

19. How many gallons are there in 12 quarts? In 16 quarts? In 8 quarts?

20. Copy and complete: $10 + 6 = ?$ $9 + 7 = ?$ $16 - 9 = ?$ $16 - 8 = ?$ $12 + 4 = ?$ $16 - 6 = ?$ $16 - 7 = ?$ $8 + 8 = ?$ $7 + 9 = ?$ $16 - 4 = ?$

21. Copy and complete: $2 \times 8 = ?$ $16 \div 2 = ?$ $4 \times 4 = ?$ $16 \div 4 = ?$ $8 \times 2 = ?$ $16 \div 8 = ?$

22. Multiply:

2	4	8	4	5	6
8	4	2	3	2	2

23. Divide: $2)\underline{16}$; $4)\underline{16}$; $8)\underline{16}$; $4)\underline{12}$.

LESSON X.

The Number Seventeen.

1. Take 17 sticks. Separate them into 10 and 7. Into 11 and 6. Into 9 and 8.

2. Show that 17 equals 12 and 5; 11 and 6; 10 and 7; 9 and 8.

3. How many are 10 and 7? 7 from 17? 7 and 10? 10 from 17? 9 and 8? 8 from 17? 8 and 9?

4. There were 10 boys and 7 girls in a class; how many pupils in the class?

5. In a class of 17 pupils 6 boys and 4 girls were absent; how many were present?

6. There were 17 bananas in a bunch; after 7 were sold, how many were left?

7. A foot and how many inches more will make 17 inches? 17 are how many more than 5?

8. Take 17 pieces of paper and imagine they were little birds, and that 8 of them flew away; how many would be left?

9. A dozen and 5 oranges make how many oranges? How many cents in a dime and 5 cents?

10. A newsboy bought 17 papers, and sold a dozen of them; how many had he left?

11. If you had a dime, how many more cents must I give you to make 17 cents?

12. I have three 5-cent pieces; how many more cents will it take to make 17 cents?

13. Carrie had 8 roses and Fannie had 9 roses; how many roses did they both have?

14. A dime, a nickel, and how many cents more will make 17 cents?

15. How many pints are there in 7 quarts and 3 pints? In 8 quarts and 2 pints?

16. Copy and add:

	10	9	9	8	9	8
	7	8	6	9	5	7

17. Copy and subtract:

	17	17	17	17	17	17
	8	10	9	6	11	12

LESSON XI.

The Number Eighteen.

1. Take 18 sticks and separate them into 9 and 9.

2. How many are 9 and 9? How many are 9 from 18?

3. If Peter spelled 9 words in the forenoon and 9 in the afternoon, how many words did he spell in all?

4. Mary's rosebush bore 18 roses, and she picked off 9 for her sister; how many roses remained on the bush?

5. How many are nine 2's? Six 3's? Three 6's? Two 9's? 2 times 9? 3 times 6?

6. Separate 18 into 2's. How many 2's make 18? How many 2's in 18?

7. Separate 18 into 3's. How many 3's make 18? How many 3's in 18?

8. If a copy of the "Times" costs 2 cents, how much would 9 copies cost?

9. How many 6's make 18? How many 6's in 18? How many 9's in 18?

10. At 2 cents each, how many pencils can I buy for 18 cents? For 12 cents?

11. If I paid 3 cents apiece for 6 newspapers, what did they all cost me?

12. Bertha spent 18 cents for 3-cent postage-stamps; how many did she buy?

13. How many feet in 6 yards? How many yards in 18 feet? In 12 feet?

14. What will 3 pounds of fish cost at 6 cents a pound?

15. How many cantelopes will 18 cents buy at 6 cents apiece? At 9 cents apiece?

16. How many horns will 9 cows have? How many are 9 times 2? 6 times 3? 3 times 6?

17. If 2 yards of muslin cost 18 cents, what will one yard cost? What is one-half of 18 cents?

18. If 3 oranges cost 18 cents, what will one orange cost? What is one-third of 18 cents?

19. Copy and add:

9	6	5	4	3	2	10	11	12	13
9	12	13	14	15	16	8	7	6	5

20. Copy and subtract:

18	18	18	18	18	18	18	18	18	18
9	8	6	7	5	3	4	10	12	15

21. Copy and multiply:

2	3	6	9	5
9	6	3	2	5

22. Copy and divide: 2)18; 3)18; 6)18; 9)18.

LESSON XII.

The Number Nineteen.

1. With objects show how many are 17 and 2; 16 and 3; 15 and 4; 14 and 5.

2. With objects show how many are 2 from 19; 3 from 19; 4 from 19; 5 from 19.

3. If I have 16 cents, how many more must I have to make 19?

4. Lucy wrote 12 lines in the forenoon and 7 lines in the afternoon; how many lines did she write?

5. Phœbe had 19 little chickens, and 8 of them died; how many remained?

6. A boy started to walk 19 miles; after walking 6 miles, how far had he yet to walk?

7. A dime, a 5-cent piece, and how many cents more will make 19 cents?

8. Two 5-cent pieces and how many cents more will make 19 cents?

9. Copy and complete: $10 + 9 = ?$ $11 + 8 = ?$ $13 + 6 = ?$ $12 + 7 = ?$ $14 + 5 = ?$

10. Copy and complete: $19 - 6 = ?$ $19 - 8 = ?$ $19 - 7 = ?$ $19 - 10 = ?$ $19 - 15 = ?$

11. Add:

4	6	6	6	9	4	2	5	4	3
8	4	5	3	3	3	4	4	2	2
1	9	7	8	7	12	11	10	13	14

12. Subtract:

19	19	19	19	19	19	19	19	19	19
10	8	12	7	11	9	6	5	14	16

13. Copy and complete:

$17 + 2 = ?$ $16 + 3 = ?$ $12 + 6 = ?$ $8 \times 2 = ?$
$19 - 4 = ?$ $14 - 7 = ?$ $18 - 8 = ?$ $6 \times 3 = ?$

LESSON XIII.

The Number Twenty.

1. Take 20 sticks and separate them into 10 and 10.

2. How many tens make 20? Two 10's are how many? Two times 10 are how many?

3. Show with blocks that 20 equals 11 and 9; 12 and 8; 13 and 7; 10 and 10.

4. Show with blocks how many are 14 and 6; 15 and 5; 16 and 4; 17 and 3.

5. How many are 11 and 9? 9 from 20? 8 and 12? 8 from 20? 13 and 7? 7 from 20? 14 and 6? 6 from 20? 15 and 5? 5 from 20? 16 and 4?

6. There were 20 cars in a station, and 12 were taken away; how many cars were left?

7. Frank learned 9 verses, and Alice learned 11 verses; how many verses did they both learn?

8. How many are ten 2's? How many are five 4's? How many are four 5's?

9. How many 2's in 20? How many 4's in 20? How many 5's in 20? How many 10's in 20?

10. What is one-half of 20? One-fourth of 20?

11. A newsboy bought 20 papers, and sold 14 of them; how many did he then have?

12. A dime and how many cents will make 20 cents?

13. A dime and how many nickels will make 20 cents?

14. A dozen apples and how many more will make 20 apples?

15. Copy and complete: $16 + 4 = ?$ $15 + 5 = ?$ $14 + 6 = ?$ $13 + 7 = ?$ $12 + 8 = ?$ $20 - 5 = ?$ $20 - 4 = ?$ $20 - 8 = ?$ $20 - 7 = ?$ $20 - 9 = ?$

16. Copy and complete: $2 \times 10 = ?$ $4 \times 5 = ?$ $5 \times 4 = ?$ $10 \times 2 = ?$ $20 \times 1 = ?$ $20 \div 2 = ?$ $20 \div 4 = ?$ $20 \div 5 = ?$ $20 \div 10 = ?$ $20 \div 1 = ?$

LESSON XIV.

Review of Elementary Results to Twenty.

1. Mary had a half-dime and 3 cents; how many cents had she?

2. Sarah is 12 years old; how old will she be in 6 more years?

3. How many inches must I add to 7 inches to make a foot?

4. How many inches long is a string which is a foot and a half long?

5. How many panes of glass in 4 windows if there are 6 panes in 1 window?

6. If a quart of milk costs 10 cents, what will a pint of milk cost?

7. If a pint of milk costs 2 cents, what will 2 quarts of milk cost?

8. If a pair of shoes cost 4 dollars, how many pairs can you buy for 20 dollars?

9. If I had 6 two-cent pieces and spent 4 cents, how much should I have remaining?

10. How many months in a year? How many months in a year and a half?

11. How many 5-cent stamps can you buy for 20 cents?

12. In a class of 18 pupils, half of them are standing and half are sitting; how many are standing?

13. If 16 sheep were in a field, and one-fourth of them jumped out, how many remained?

14. Florence has a half-dime and three 2-cent pieces; how many cents has she?

15. Jane had 2 dimes, and spent them for crayons at 2 cents each; how many crayons did she buy? How many pencils at 4 cents each could she have bought?

16. How many two-cent stamps can I buy with a dime and 4 cents?

17. How many feet have 4 cows? How many horns have 6 oxen?

18. How many horses can be shod with 12 shoes? How many with 16 shoes?

19. Edith bought a dozen and 6 oranges; how many did she buy?

20. Carrie had a dime and a nickel, and spent 8 cents for her lunch; how many cents remained?

21. How many sides have 6 triangles? How many sides have 5 squares?

22. How many triangles can you make out of 18 little sticks?

23. How many squares can you make out of 20 little sticks?

LESSON XV.

Inexact Divisors.

1. Show with blocks how many 2's in 5, and how many remain.

2. Show how many 2's in 3. In 7. In 9. In 11.

3. How many apples at 2 cents each can I buy for 7 cents?

4. How much will be left out of 5 cents if I buy two 2-cent stamps?

5. Show with sticks how many 3's in 5, and how many remain.

6. Show how many 3's in 7. In 8. In 9. In 10. In 11. In 13.

7. How many 3-cent stamps can I buy for 11 cents? How many cents will remain?

8. How many triangles can I make with 14 little sticks? How many sticks remain?

9. How many 4's in 6? In 9? In 11? In 14? In 17? In 15? In 18?

10. How many squares can I make with 10 little sticks?

11. Thirteen pints of wine will fill how many quart bottles?

12. How many 5's in 8? In 11? In 13? In 17? In 20? In 16? In 18?

13. How many 5-cent stamps can I buy for 12 cents?

14. How many Sunday papers, at 5 cents each, can you buy for 16 cents?

15. How many 6's in 10? In 14? In 17? In 19? In 20? In 15? In 18?

16. At 6 cents each, how many pies can I buy for 14 cents?

WRITTEN EXERCISES.

Copy and write the results of the following:

$7 + 4$	$15 - 7$	$8 + 6$	$15 - 9$	$7 + 8$
$9 + 6$	$16 - 5$	$7 + 9$	$18 - 7$	$6 + 9$
$8 + 7$	$17 - 8$	$9 + 8$	$17 - 6$	$9 + 7$
$7 + 6$	$14 - 9$	$8 + 9$	$19 - 9$	$10 + 8$
9×3	$11 \div 5$	$12 \div 7$	$8 \div 8$	$9 \div 9$
8×4	$15 \div 6$	$14 \div 7$	$10 \div 8$	$12 \div 9$
$20 \div 5$	$17 \div 3$	$16 \div 7$	$17 \div 8$	$18 \div 9$
$16 \div 8$	$18 \div 4$	$18 \div 5$	$20 \div 8$	$20 \div 9$
$4)\underline{12}$	$5)\underline{12}$	$6)\underline{14}$	$7)\underline{18}$	$8)\underline{20}$

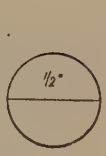
SIGHT EXERCISES.

Give answers:

$8 + 5$	$7 + 4$	$5 + 8$	$10 \div 5$	10×2
$9 - 6$	8×2	$9 - 7$	$12 \div 4$	$18 \div 6$
5×3	$9 \div 3$	6×3	$16 \div 8$	$15 \div 4$
4×5	7×2	$9 \div 4$	$17 \div 5$	$12 \div 5$

LESSON XVI.

Lesson on Fractional Parts.



1. What do we call each part of this circle?

2. What is one-half of anything?

3. *One-half* is written thus: $\frac{1}{2}$.

4. What is $\frac{1}{2}$ of 4? $\frac{1}{2}$ of 6? $\frac{1}{2}$ of 8? $\frac{1}{2}$ of 10? $\frac{1}{2}$ of 16? $\frac{1}{2}$ of 14? $\frac{1}{2}$ of 18?

5. If I have 12 apples, and give $\frac{1}{2}$ of them to my sister, how many do I give to my sister?

6. If Edith has 14 roses, and gives $\frac{1}{2}$ of them to Frank, how many roses then has Edith?

7. What do we call each part of this circle?

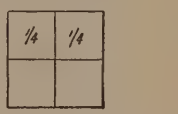
8. What is one-third of anything?

9. We write *one-third* thus: $\frac{1}{3}$.

10. What is $\frac{1}{3}$ of 6? $\frac{1}{3}$ of 9? $\frac{1}{3}$ of 12? $\frac{1}{3}$ of 15?

11. My father had 6 horses, and sold $\frac{1}{3}$ of them; how many did he sell?

12. My brother had 12 marbles, and gave me $\frac{1}{3}$ of them; how many did he give me?



13. What do we call each part of this square?

14. What is one-fourth of anything?

15. We write *one-fourth* thus: $\frac{1}{4}$.

16. What is $\frac{1}{4}$ of 8? $\frac{1}{4}$ of 12? $\frac{1}{4}$ of 16? $\frac{1}{4}$ of 20?

17. If 4 boys share 16 marbles equally, how many marbles will each have?

18. *One-fifth* of anything is written thus: $\frac{1}{5}$.

19. What is $\frac{1}{5}$ of 5? $\frac{1}{5}$ of 10? $\frac{1}{5}$ of 15? $\frac{1}{5}$ of 20?

20. If 5 apples cost 15 cents, what will one apple cost?

21. *One-sixth* of anything is written thus: $\frac{1}{6}$.

22. What is $\frac{1}{6}$ of 6? $\frac{1}{6}$ of 12? $\frac{1}{6}$ of 18?

23. Copy and complete:

$\frac{1}{2}$ of 8 = ? $\frac{1}{3}$ of 9 = ? $\frac{1}{4}$ of 12 = ? $\frac{1}{5}$ of 10 = ?
$\frac{1}{2}$ of 12 = ? $\frac{1}{3}$ of 12 = ? $\frac{1}{4}$ of 16 = ? $\frac{1}{5}$ of 20 = ?
$\frac{1}{2}$ of 14 = ? $\frac{1}{3}$ of 18 = ? $\frac{1}{4}$ of 20 = ? $\frac{1}{6}$ of 18 = ?

LESSON XVII.

Expressing Numbers from Twenty to One Hundred.

1. Write 2 tens and 1, or *twenty-one.* *Ans.* 21.

2. Write 2 tens and 2, or *twenty-two.* *Ans.* 22.

3. Write 2 tens and 3, or *twenty-three.* *Ans.* 23.

4. What does each figure denote in 23? In 21? In 20?

5. Two tens and four units are *twenty-four*, written 24.

6. Two tens and five units are *twenty-five*, written 25.

7. Two tens and six units are *twenty-six*, written 26.

8. Two tens and seven units are *twenty-seven*, written 27.

9. Two tens and eight units are *twenty-eight*, written 28.

10. Two tens and nine units are *twenty-nine*, written 29.

11. Three tens are *thirty*, written 30.

12. Write the numbers from Thirty to Thirty-nine.

13. Four tens are *forty*, written 40.

14. Write the numbers from Forty to Forty-nine.

15. Five tens are *fifty*, written 50.

16. Write the numbers from Fifty to Fifty-nine.

17. Six tens are *sixty*, written 60.

18. Write the numbers from Sixty to Sixty-nine.

19. Seven tens are *seventy*, written 70.

20. Write the numbers from Seventy to Seventy-nine.

21. Eight tens are *eighty*, written 80.

22. Write the numbers from Eighty to Eighty-nine.

23. Nine tens are *ninety*, written 90.

24. Write the numbers from Ninety to Ninety-nine.

25. If we make a bunch of sticks, consisting of ten tens, we call it *one hundred.*

We write one hundred thus: 100.

NOTE.—The teacher will drill pupils on writing numbers up to 1000.

SECTION III.

ELEMENTARY RESULTS.

LESSON I.

Adding and Subtracting with Two.

1. How many are 1 and 2? 2 and 2? 4 and 2? 3 and 2? 5 and 2? 6 and 2?

2. How many are 7 and 2? 9 and 2? 8 and 2? 10 and 2? 11 and 2? 12 and 2?

3. We will now write the results we have obtained in a little table:

ADDITION TABLE.

$1 + 2 = 3.$	$4 + 2 = 6.$	$7 + 2 = 9.$	$10 + 2 = 12.$
$2 + 2 = 4.$	$5 + 2 = 7.$	$8 + 2 = 10.$	$11 + 2 = 13.$
$3 + 2 = 5.$	$6 + 2 = 8.$	$9 + 2 = 11.$	$12 + 2 = 14.$

4. Ella had 3 pins and found 2 pins; how many pins did Ella then have?

5. How many are 2 taken from 3? 2 from 4? 2 from 6? 2 from 8? 2 from 10?

6. How many are 2 taken from 5? 2 from 7? 2 from 9? 2 from 11? 2 from 12?

7. We will now write the results we have obtained in a little table:

SUBTRACTION TABLE.

$3 - 2 = 1.$	$6 - 2 = 4.$	$9 - 2 = 7.$	$12 - 2 = 10.$
$4 - 2 = 2.$	$7 - 2 = 5.$	$10 - 2 = 8.$	$13 - 2 = 11.$
$5 - 2 = 3.$	$8 - 2 = 6.$	$11 - 2 = 9.$	$14 - 2 = 12.$

8. Willie had 6 marbles, and lost 2 of them; how many marbles did Willie then have?

9. Begin at 2, and count by 2's to 14. Begin at 1, and count by 2's to 13.

10. Begin at 14, and count backward by 2's to 2. Begin at 13, and count backward by 2's to 1.

LESSON II.

Multiplying and Dividing with Two.

1. How many are 2 times 2? 3 times 2? 4 times 2? 5 times 2?

2. How many are 6 times 2? 7 times 2? 8 times 2? 9 times 2?

3. We will now write the results we have obtained in a little table:

MULTIPLICATION TABLE.

2 times 2 are 4.	6 times 2 are 12.
3 times 2 are 6.	7 times 2 are 14.
4 times 2 are 8.	8 times 2 are 16.
5 times 2 are 10.	9 times 2 are 18.

4. At 2 cents each, what will a bunch of 5 pinks cost?

5. If 1 ox has 2 horns, how many horns have 6 oxen?

6. How many 2's in 4? 2's in 6? 2's in 8? 2's in 10? 2's in 12? 2's in 14? 2's in 16? 2's in 18?

7. We will now write the results we have obtained in a little table:

DIVISION TABLE.

$4 \div 2 = 2$. $8 \div 2 = 4$. $12 \div 2 = 6$. $16 \div 2 = 8$.
$6 \div 2 = 3$. $10 \div 2 = 5$. $14 \div 2 = 7$. $18 \div 2 = 9$.

8. Ethel paid 10 cents for 2-cent postage-stamps; how many did she buy?

9. Effie paid 14 cents for buttons at 2 cents each; how many did she buy?

WRITTEN EXERCISES.

Multiply:

2	2	2	2	2	21	22	22	21	21	21	21
5	7	6	9	10	3	4	3	5	6	7	8

Divide:

2)10 2)14 2)16 2)18 2)20 2)22 2)24 2)26

NOTE.—The teacher will explain how to multiply and divide when the examples transcend the limits of the "Elementary Results."

LESSON III.

Adding and Subtracting with Three.

1. How many are 4 and 3? 6 and 3? 5 and 3? 7 and 3? 9 and 3? 8 and 3? 11 and 3? 10 and 3? 12 and 3?

2. Ella had 7 pins, and found 3 pins; how many pins did Ella then have?

3. Mary saw 8 robins in the garden and 3 robins on the fence; how many robins did she see?

4. How many are 3 from 7? 3 from 9? 3 from 11? 3 from 10? 3 from 12? 3 from 14? 3 from 15?

5. Alice wrote 7 words on her slate, and rubbed out 3 of them; how many words were left?

6. There were 10 birds on a tree, and 3 of them flew away; how many birds remained?

7. Add by 3's from 3 to 15. Add by 3's from 1 to 13. Add by 3's from 2 to 14.

8. Subtract by 3's from 15 to naught; from 14 to 2; from 13 to 1.

WRITTEN EXERCISES.

Add:

3	3	3	3	3	3	3	3	3	3	3	3	3
6	5	4	8	10	9	11	7	12	14	13	16	15

23	31	32	13	23	30	32	68	31	54	73	85	94
46	28	65	73	64	37	54	31	46	33	23	23	33

From	7	9	8	10	12	11	14	15	13	16	17	19	18
Take	3	3	3	3	3	3	3	3	3	3	3	3	3

From	57	63	72	84	45	77	89	96	95
Take	23	32	21	13	22	31	32	33	30

NOTE.—The teacher will show the pupil how to add and subtract when the examples transcend the "Elementary Results."

LESSON IV.

Multiplying and Dividing with Three.

1. How many are 2 times 3? 3 times 3? 4 times 3? 5 times 3? 6 times 3? 7 times 3? 8 times 3? 9 times 3?

2. We will write the results we have obtained in a little table, which the pupil will try to remember:

MULTIPLICATION TABLE.

$3 \times 2 = 6$. $3 \times 4 = 12$. $3 \times 6 = 18$. $3 \times 8 = 24$.
$3 \times 3 = 9$. $3 \times 5 = 15$. $3 \times 7 = 21$. $3 \times 9 = 27$.

3. Mary bought 7 yards of ribbon at 3 cents a yard; how much did it cost?

4. Jane bought 10 pencils at 3 cents each; how much did they cost her?

5. How many 3's in 6? 3's in 9? 3's in 12? 3's in 15? 3's in 18? 3's in 21? 3's in 24? 3's in 27?

6. We will write the results we have obtained in a little table, which the pupil will try to remember:

DIVISION TABLE.

$6 \div 3 = 2$. $12 \div 3 = 4$. $18 \div 3 = 6$. $24 \div 3 = 8$.
$9 \div 3 = 3$. $15 \div 3 = 5$. $21 \div 3 = 7$. $27 \div 3 = 9$.

7. If you pay 12 cents for melons, at 3 cents apiece, how many can you buy?

8. How many 3-cent postage-stamps can you buy for 18 cents?

WRITTEN EXERCISES.

Multiply:

3	3	3	3	23	31	32	30	31	31	30
7	8	9	10	2	3	4	5	6	8	9
23	31	32	31	30	31	31	31	32	32	23
3	4	3	5	6	7	9	2	4	4	3

Divide:

3)9 3)12 3)15 3)18 3)21 3)24 3)27 3)30

LESSON V.

Adding and Subtracting with Four.

1. How many are 3 and 4? 6 and 4? 5 and 4? 7 and 4? 10 and 4? 9 and 4? 8 and 4? 11 and 4? 12 and 4?

2. If there are 7 boys on one bench, and 4 boys on another bench, how many boys on both benches?

3. I gave 9 cents to a poor woman, and 4 cents to her little daughter; how much did I give to both?

4. Write the Addition Table for 4's on the blackboard or slate, using the signs + and =.

5. Add by 4's from 4 to 16; from 1 to 13; from 2 to 14; from 3 to 15.

6. How many are 4 from 7? 4 from 9? 4 from 8? 4 from 11? 4 from 13? 4 from 12? 4 from 10? 4 from 14? 4 from 16?

7. Ethel had 10 roses, and gave her mother 4 roses; how many roses did Ethel keep?

8. Twelve boys were playing ball, and 4 of them left; how many boys remained?

9. Count backward by 4's from 16 to 0; from 15 to 3; from 14 to 2; from 13 to 1.

WRITTEN EXERCISES.

Add:

4	4	4	4	4	4	4	4	4	4	4	4	4
6	5	9	8	10	12	11	13	15	14	16	18	20
43	41	14	23	33	43	42	44	64	52	63	83	96
45	36	72	66	54	56	37	55	45	34	24	34	23

From	7	9	11	13	15	16	18	19	20	23	24
Take	4	4	4	4	4	4	4	4	4	4	4
From	65	79	87	99	58	69	75	98	76	109	118
Take	41	34	23	43	22	43	24	14	13	47	44

LESSON VI.

Multiplying and Dividing with Four.

1. How many are 2 times 4? 3 times 4? 4 times 4? 5 times 4? 6 times 4? 7 times 4? 8 times 4? 9 times 4?

2. We will now write the results we have obtained in a little table, which the pupil will try to remember:

MULTIPLICATION TABLE.

$4 \times 2 = 8$. $4 \times 4 = 16$. $4 \times 6 = 24$. $4 \times 8 = 32$.
$4 \times 3 = 12$. $4 \times 5 = 20$. $4 \times 7 = 28$. $4 \times 9 = 36$.

3. If a boy works 4 examples a day, how many will he work in 6 days?

4. If 1 loaf of bread costs 4 cents, how many cents will 10 loaves of bread cost?

5. How many 4's in 8? 4's in 12? 4's in 16? 4's in 20? 4's in 24? 4's in 28? 4's in 32? 4's in 36?

6. If a bird flies a mile in 4 minutes, how far will it fly in 12 minutes?

7. If a man walks 4 miles in 1 hour, how long will it take him to walk 20 miles?

8. Fill out the following on the slate or blackboard:

$8 \div 4 = ?$ $16 \div 4 = ?$ $24 \div 4 = ?$ $32 \div 4 = ?$
$12 \div 4 = ?$ $20 \div 4 = ?$ $28 \div 4 = ?$ $36 \div 4 = ?$

WRITTEN EXERCISES.

Multiply:

4	4	14	42	41	40	41	44	40	41	40
8	9	2	3	4	5	6	2	8	9	7
43	42	41	40	41	42	41	40	42	42	41
3	4	5	6	7	3	8	9	4	7	6

Divide:

4)8	4)12	4)16	4)20	4)24	4)28	4)32	4)36
4)40	4)44	4)48	4)84	4)88	4)80	4)120	4)124

LESSON VII.

Adding and Subtracting with Five.

1. How many are 3 and 5? 5 and 5? 4 and 5? 6 and 5? 7 and 5? 9 and 5? 8 and 5? 10 and 5? 11 and 5? 12 and 5?

2. Mary has 6 books at school, and 5 books at home; how many books has she?

3. James has 8 cents in a box, and 5 cents in his pocket; how many cents has he?

4. Add by 5's from 5 to 20; from 1 to 16; from 2 to 17; from 3 to 18; from 4 to 19.

5. How many are 5 from 8? 5 from 10? 5 from 9? 5 from 11? 5 from 13? 5 from 15? 5 from 14? 5 from 16? 5 from 18?

6. James is 9 years old, and his sister is 5 years younger; how old is his sister?

7. Ada culled 14 roses, and gave Willis 5 of them; how many roses did Ada retain?

8. Write the Addition Table for 5's. Write the Subtraction Table for 5's.

WRITTEN EXERCISES.

Add:	5	5	5	5	5	5	5	5	5	5	5	5
	6	8	9	10	11	15	17	18	20	22	24	25
Add:	54	45	53	25	55	54	35	45	45	35	65	74
	33	71	62	44	23	25	74	77	86	47	38	48
From	9	11	10	12	13	15	18	20	24	26	28	30	34
Take	5	5	5	5	5	5	5	5	5	5	5	5	5
From	69	84	79	97	81	91	99	76	67	74	88	86	98
Take	54	52	45	54	40	51	45	52	45	52	55	52	55

NOTE.—The teacher will show the pupils how to "carry" in the second row of examples in Addition.

LESSON VIII.

Multiplying and Dividing with Five.

1. How many are 2 times 5? 3 times 5? 4 times 5? 5 times 5? 6 times 5? 7 times 5? 8 times 5? 9 times 5?

2. Fill out the following and commit to memory:

$5 \times 2 = ?$ $5 \times 4 = ?$ $5 \times 6 = ?$ $5 \times 8 = ?$
$5 \times 3 = ?$ $5 \times 5 = ?$ $5 \times 7 = ?$ $5 \times 9 = ?$

3. How much must 6 little boys pay to ride to school, if the fare of each is 5 cents?

4. If a boy has 5 fingers on each hand, how many fingers have 4 boys?

5. How many 5's in 10? 5's in 15? 5's in 20? 5's in 25? 5's in 30? 5's in 35? 5's in 40? 5's in 45?

6. How many pairs of shoes, at 5 dollars a pair, can you buy for 30 dollars?

7. How many books can I buy for fifty cents, at the rate of 5 cents apiece?

8. Complete the following:

$10 \div 5 = ?$ $20 \div 5 = ?$ $30 \div 5 = ?$ $40 \div 5 = ?$
$15 \div 5 = ?$ $25 \div 5 = ?$ $35 \div 5 = ?$ $45 \div 5 = ?$

WRITTEN EXERCISES.

Multiply:

5	5	5	5	23	54	52	52	53	51	50	51
6	8	7	9	3	2	3	4	3	6	7	8
15	25	35	45	53	54	45	65	58	35	25	
3	2	3	2	4	3	5	3	4	6	8	

Divide:

5)10	5)15	5)20	5)25	5)30	5)35	5)40	5)45
5)50	5)55	5)100	5)105	5)150	5)155	5)200	5)205

NOTE.—The teacher will show the pupils how to "carry" in the second row of problems in Multiplication.

LESSON IX.

Adding and Subtracting with Six.

1. How many are 4 and 6? 6 and 6? 5 and 6? 7 and 6? 9 and 6? 11 and 6? 8 and 6? 10 and 6? 12 and 6?

2. Edith had 7 pins in her cushion, and put in 6 more; how many pins were there in the cushion?

3. There are 10 birds on a tree and 6 birds on the ground; how many birds in all?

4. Count forward by 6's from 6 to 18; from 5 to 17; from 4 to 16; from 3 to 15; from 2 to 14.

5. Six from 8 leaves how many? 6 from 9? 6 from 11? 6 from 10? 6 from 12? 6 from 13? 6 from 15? 6 from 17? 6 from 18?

6. Maria bought 11 apples, and gave Edward 6 of them; how many apples did Maria keep?

7. If you make 17 marks on your slate, and rub out 6 of them, how many marks will remain?

8. Count backward by 6's from 18; from 17 to 5; from 16 to 4; from 15 to 3.

9. Write the Addition Table for 6's. Write the Subtraction Table for 6's,

WRITTEN EXERCISES.

Add:	6	6	6	6	6	6	6	6	6	6	6	6	6
	5	7	6	8	10	9	11	13	12	14	16	18	20
Add:	26	46	56	62	45	36	65	54	46	56	75	84	96
	43	32	21	34	64	92	53	64	74	67	46	28	34
From	9	10	13	11	14	15	18	16	19	20	22	24	26
Take	6	6	6	6	6	6	6	6	6	6	6	6	6

From	94	75	87	89	93	107	128	156	128	145
Take	63	61	66	56	60	63	64	64	56	63

NOTE.—The teacher will show pupils how to "carry" in Addition, and how to "borrow" in Subtraction.

LESSON X.

Multiplying and Dividing with Six.

1. How many are 2 times 6? 3 times 6? 4 times 6? 5 times 6? 6 times 6? 7 times 6? 8 times 6? 9 times 6?

2. Fill out the following and commit to memory:

$6 \times 2 = ?$ $6 \times 4 = ?$ $6 \times 6 = ?$ $6 \times 8 = ?$
$6 \times 3 = ?$ $6 \times 5 = ?$ $6 \times 7 = ?$ $6 \times 9 = ?$

3. If Henry learns 6 lessons a day, how many lessons will he learn in 6 days?

4. If a street-car goes six miles in 1 hour, how many miles will it go in 8 hours?

5. How many times 6 in 12? 6 in 18? 6 in 24? 6 in 30? 6 in 36? 6 in 42? 6 in 48? 6 in 54?

6. If Mary's doll cost 6 cents, how many dolls could she buy for 30 cents?

7. If a boat sails 6 miles an hour, how far will it sail in 36 hours?

8. Complete the following:

$12 \div 6 = ?$ $24 \div 6 = ?$ $36 \div 6 = ?$ $48 \div 6 = ?$
$18 \div 6 = ?$ $30 \div 6 = ?$ $42 \div 6 = ?$ $54 \div 6 = ?$

WRITTEN EXERCISES.

Multiply:

6	6	6	6	6	62	63	60	61	60	64
5	7	8	9	10	4	3	6	5	9	2
24	35	26	36	56	52	46	43	64	32	43
3	4	5	4	3	6	5	8	6	9	7

Divide:

6)12	6)18	6)24	6)30	6)36	6)42	6)48	6)54
6)60	6)66	6)120	6)126	6)180	6)186	6)240	6)246

NOTE.—The teacher will show the pupils how to "carry" in the second row of problems in Multiplication.

LESSON XI.

Adding and Subtracting with Seven.

1. How many are 2 and 7? 4 and 7? 6 and 7? 5 and 7? 3 and 7? 7 and 7? 8 and 7? 10 and 7? 9 and 7? 11 and 7? 12 and 7? 13 and 7?

2. There are 8 birds on one tree and 7 birds on another tree; how many birds are there on both trees?

3. There are 9 sheep in one field and 7 sheep in another field; how many sheep are in both fields?

4. How many are 7 from 9? 7 from 10? 7 from 12? 7 from 11? 7 from 13? 7 from 14? 7 from 16? 7 from 18? 7 from 15? 7 from 17? 7 from 19?

5. There were 10 cows in a field, and 7 of them jumped out; how many cows remained in the field?

6. Jane culled 7 roses from a bush containing 15 roses; how many roses remained on the bush?

7. Francis had 5 cents, and found 4 cents, and then spent 6 cents; how many cents remained?

8. Write the Addition Table for 7's. Write the Subtraction Table for 7's.

WRITTEN EXERCISES.

Add:

7	7	7	7	7	7	7	7	7	7	7	7	7
6	12	13	15	17	18	20	22	23	25	27	30	32
37	57	67	74	76	77	76	57	75	76			
42	72	54	54	87	45	57	95	63	64			

Subtract:

15	18	16	17	20	23	25	27	30	32	34	36	40
7	7	7	7	7	7	7	7	7	7	7	7	7
48	59	89	90	125	136	147	158	208	247			
37	47	75	70	75	74	75	76	77	75			

NOTE.—Show the pupils how to "carry" in adding, and how to "borrow" in subtracting.

LESSON XII.

Multiplying and Dividing with Seven.

1. How many are 2 times 7? 3 times 7? 4 times 7? 5 times 7? 6 times 7? 7 times 7? 8 times 7? 9 times 7?

2. Fill out the following and commit to memory:

$7 \times 2 = ?$ $7 \times 4 = ?$ $7 \times 6 = ?$ $7 \times 8 = ?$
$7 \times 3 = ?$ $7 \times 5 = ?$ $7 \times 7 = ?$ $7 \times 9 = ?$

3. There are 7 days in 1 week; how many days are there in 4 weeks?

4. If a man can row a boat 7 miles in 1 hour, how far can he row in 6 hours?

5. How many times 7 in 14? 7 in 21? 7 in 28? 7 in 35? 7 in 42? 7 in 49? 7 in 56? 7 in 63?

6. If 1 line of printing contains 7 words, how many lines of printing will 28 words make?

7. There are 7 days in 1 week; how many weeks are there in 42 days? In 56 days?

8. Complete the following:

$14 \div 7 = ?$ $28 \div 7 = ?$ $42 \div 7 = ?$ $56 \div 7 = ?$
$21 \div 7 = ?$ $35 \div 7 = ?$ $49 \div 7 = ?$ $63 \div 7 = ?$

WRITTEN EXERCISES.

Multiply:

7	7	7	7	7	71	70	72	73	74	71
5	6	7	8	9	7	8	4	3	2	6

23	25	27	37	47	57	67	77	57	76
4	3	2	3	4	5	4	3	2	5

Divide:

7)14	7)21	7)28	7)35	7)42	7)49	7)56	7)63
7)70	7)77	7)140	7)147	7)210	7)217	7)280	7)287

NOTE.—The teacher will show the pupils how to "carry" in the second row of examples in Multiplication.

LESSON XIII.

Adding and Subtracting with Eight.

1. How many are 1 and 8? 3 and 8? 2 and 8? 5 and 8? 4 and 8? 6 and 8? 7 and 8? 10 and 8? 12 and 8? 9 and 8? 11 and 8?

2. Laura has 7 credit-marks, and her brother has 8; how many credit-marks have both?

3. A cat caught 5 mice one night and 8 mice the next night; how many did she catch both nights?

4. How many are 8 from 10? 8 from 12? 8 from 9? 8 from 11? 8 from 13? 8 from 16? 8 from 14? 8 from 17? 8 from 19? 8 from 18? 8 from 20?

5. If there are 10 swallows on the barn, and 8 of them fly away, how many swallows remain?

6. There were 12 persons at dinner, and 8 of them left the table; how many persons remained?

7. Mary had 12 cents, and lost 4 cents, and then found 6 cents; how many cents had she then?

8. Write the Addition Table for 8's. Write the Subtraction Table for 8's.

WRITTEN EXERCISES.

Add:

8	8	8	8	8	8	8	8	8	8	8	8	8
7	9	10	12	14	16	28	22	25	26	30	31	34
85	78	38	82	85	88	86	83	87	88			
34	61	41	46	64	75	73	80	75	84			

Subtract:

12	15	14	17	19	20	24	26	30	32	39	40	42
8	8	8	8	8	8	8	8	8	8	8	8	8
99	97	124	136	148	164	172	173	197	208			
38	82	85	82	87	85	86	85	84	86			

NOTE.—The teacher will show the pupils how to "carry" in Addition, and how to "borrow" in Subtraction.

LESSON XIV.

Multiplying and Dividing with Eight.

1. How many are 2 times 8? 3 times 8? 4 times 8? 5 times 8? 6 times 8? 7 times 8? 8 times 8? 9 times 8?

2. Fill out the following and commit to memory:

$8 \times 2 = ?$ $8 \times 4 = ?$ $8 \times 6 = ?$ $8 \times 8 = ?$
$8 \times 3 = ?$ $8 \times 5 = ?$ $8 \times 7 = ?$ $8 \times 9 = ?$

3. If you have 8 bones in each wrist, how many bones have you in both wrists?

4. If there are 8 quarts in 1 peck, how many quarts are there in 5 pecks?

5. How many times 8 in 16? 8 in 24? 8 in 32? 8 in 40? 8 in 48? 8 in 56? 8 in 64? 8 in 72?

6. How many tops can James buy for 32 cents, at the rate of 8 cents apiece?

7. If a yacht sails 8 miles in an hour, how many hours will it take to sail 48 miles?

8. Complete the following:

$16 \div 8 = ?$ $32 \div 8 = ?$ $48 \div 8 = ?$ $64 \div 8 = ?$
$24 \div 8 = ?$ $40 \div 8 = ?$ $56 \div 8 = ?$ $72 \div 8 = ?$

WRITTEN EXERCISES.

Multiply:

8	8	8	8	82	83	80	81	81	84	81
5	6	7	9	4	3	6	5	9	2	7
32	27	38	25	43	47	38	48	58	68	78
6	5	4	8	5	4	6	5	4	3	5

Divide:

8)16	8)24	8)32	8)40	8)48	8)56	8)64	8)72
8)80	8)88	8)160	8)168	8)240	8)248	8)320	8)328

NOTE.—The teacher will show the pupils how to "carry" in the second row of problems in Multiplication.

LESSON XV.

Adding and Subtracting with Nine.

1. How many are 2 and 9? 3 and 9? 1 and 9? 5 and 9? 6 and 9? 7 and 9? 8 and 9? 10 and 9? 12 and 9? 9 and 9? 11 and 9?

2. There were 6 boys playing marbles, and 9 boys looking on; how many boys were there in all?

3. There were 8 boys and 9 girls in a class; how many pupils in the class?

4. How many are 9 from 11? 9 from 13? 9 from 12? 9 from 14? 9 from 16? 9 from 18? 9 from 10? 9 from 15? 9 from 17? 9 from 19? 9 from 20?

5. A boy gathered 12 pears, and sent 9 of them to his teacher; how many pears had the boy left?

6. Willie had 14 rabbits in a cage, and 9 of them jumped out; how many rabbits remained in the cage?

7. Peter gave his brother 11 marbles, and had 9 marbles remaining; how many marbles had Peter at first?

8. Write the Addition Table for 9's. Write the Subtraction Table for 9's.

WRITTEN EXERCISES.

Add:

9	9	9	9	9	9	9	9	9	9	9	9	9
7	8	10	12	14	16	20	22	25	26	30	31	34

95	79	39	94	95	99	96	93	97	99
34	60	40	46	67	70	75	80	74	80

Subtract:

12	15	14	17	19	20	24	26	30	32	39	40	42
9	9	9	9	9	9	9	9	9	9	9	9	9

89	97	105	126	138	146	163	172	182	190
49	93	94	92	94	95	96	95	96	90

NOTE.—The teacher will show the pupils how to "carry" in Addition, and how to "borrow" in Subtraction.

LESSON XVI.

Multiplying and Dividing with Nine.

1. How many are 2 times 9? 3 times 9? 4 times 9? 5 times 9? 6 times 9? 7 times 9? 8 times 9? 9 times 9?

2. Fill out the following and commit to memory:

$9 \times 2 = ?$ $9 \times 4 = ?$ $9 \times 6 = ?$ $9 \times 8 = ?$
$9 \times 3 = ?$ $9 \times 5 = ?$ $9 \times 7 = ?$ $9 \times 9 = ?$

3. If William spells 9 words in 1 day, how many words at that rate will he spell in 4 days?

4. How many pinks are blooming on 6 stems, if there are 9 pinks on each stem?

5. How many times 9 in 18? 9 in 27? 9 in 36? 9 in 45? 9 in 54? 9 in 63? 9 in 72? 9 in 81?

6. Mabel gave away 36 violets, giving 9 violets to each of her playmates; how many playmates were there?

7. How many bottles of ink can I buy for 90 cents, if I pay 9 cents a bottle?

8. Complete the following:

$18 \div 9 = ?$ $36 \div 9 = ?$ $54 \div 9 = ?$ $72 \div 9 = ?$
$27 \div 9 = ?$ $45 \div 9 = ?$ $63 \div 9 = ?$ $81 \div 9 = ?$

WRITTEN EXERCISES.

Multiply:

9	9	9	9	9	91	92	93	94	91	90
5	6	7	8	9	5	4	3	2	8	9

23	32	28	19	29	34	49	59	39	29
5	6	4	5	3	4	5	6	7	8

Divide:

9)18	9)27	9)36	9)45	9)54	9)63	9)72	9)81
9)90	9)99	9)180	9)189	9)270	9)279	9)360	9)369

NOTE.—The teacher will show the pupils how to "carry" in the second row of problems in Multiplication.

LESSON XVII.

Adding and Subtracting with Ten.

1. How many are 1 and 10? 3 and 10? 2 and 10? 4 and 10? 5 and 10? 6 and 10? 7 and 10? 9 and 10? 8 and 10? 10 and 10? 12 and 10? 11 and 10?

2. A boy shot 5 birds, and a man shot 10 birds; how many birds did they both shoot?

3. In a field there are 8 horses and 10 cows; how many animals are there in the field?

4. How many are 10 from 12? 10 from 14? 10 from 13? 10 from 15? 10 from 17? 10 from 16? 10 from 18? 10 from 20? 10 from 19? 10 from 21? 10 from 22?

5. Annie's book contained 14 pages, and her dog tore out 10 pages; how many pages remained?

6. A hunter shot 10 robins out of a flock of 20 robins; how many robins remained in the flock?

7. Edith had 6 roses, and bought 10 roses, and gave her sister 8 roses; how many roses did Edith then have?

8. Write the Addition Table for 10's. Write the Subtraction Table for 10's.

WRITTEN EXERCISES.

Add:

10	10	10	10	10	10	10	10	10	10	10	10	10	10
9	8	11	13	15	17	19	21	24	23	27	32	35	37

26	34	54	63	72	67	76	65	70	98
35	28	27	55	38	25	42	29	58	36

Subtract:

12	16	18	20	23	25	27	29	32	34	35	37	39
10	10	10	10	10	10	10	10	10	10	10	10	10

34	32	34	28	37	42	53	48	64	75
8	7	18	15	14	25	36	25	27	38

NOTE.—The teacher will show the pupils how to "carry" in Addition, and how to "borrow" in Subtraction.

LESSON XVIII.

Multiplying and Dividing with Ten.

1. How many are 2 times 10? 3 times 10? 4 times 10? 5 times 10? 6 times 10? 7 times 10? 8 times 10? 9 times 10?

2. Fill out the following and commit to memory:

$10 \times 2 = ?$ $10 \times 4 = ?$ $10 \times 6 = ?$ $10 \times 8 = ?$
$10 \times 3 = ?$ $10 \times 5 = ?$ $10 \times 7 = ?$ $10 \times 9 = ?$

3. Edgar gave 10 marbles to each of his six playmates; how many marbles did he give away?

4. How many trees are there in 8 rows of trees, if there are 10 trees in each row?

5. How many times 10 in 20? In 30? In 40? In 50? In 60? In 70? In 80? In 90? 10 in 100?

6. A boy arranged 60 bricks in piles of 10 bricks in each pile; how many piles were there?

7. There are 100 orange trees in a grove, and 10 trees in each row; how many rows of trees are there?

8. Complete the following:

$20 \div 10 = ?$ $40 \div 10 = ?$ $60 \div 10 = ?$ $80 \div 10 = ?$
$30 \div 10 = ?$ $50 \div 10 = ?$ $70 \div 10 = ?$ $90 \div 10 = ?$

WRITTEN EXERCISES.

Multiply:

10	10	10	10	10	10	10	10	24	32	42
5	6	7	8	9	10	11	12	2	3	4
23	42	39	46	74	85	79	57	37	43	
7	8	4	5	3	6	2	4	9	6	

Divide:

10)20 10)30 10)40 10)50 10)60 10)70 10)80 10)90

10)100 10)110 10)120 6)48 8)56 4)84 3)69 9)90

NOTE.—The teacher will show the pupils how to "carry" in the second row of problems in Multiplication.

LESSON XIX.

Add and Subtract with Eleven and Twelve.

1. How many are 1 and 11? 3 and 11? 2 and 11? 4 and 11? 6 and 11? 5 and 11? 7 and 11? 9 and 11? 8 and 11? 10 and 11? 12 and 11? 11 and 11?

2. There are 5 pupils in one class, and 11 in another class; how many pupils in both classes?

3. How many are 11 from 13? 11 from 15? 11 from 14? 11 from 16? 11 from 18? 11 from 19? 11 from 20? 11 from 22? 11 from 21? 11 from 23?

4. If I have a dime and a "nickel," and spend 11 cents for marbles, how many cents will remain?

5. How many are 2 and 12? 3 and 12? 4 and 12? 6 and 12? 5 and 12? 7 and 12? 9 and 12? 8 and 12? 10 and 12? 11 and 12? 12 and 12?

6. Mary has 10 cents in her pocket-book and 12 cents in her money-box; how many cents has she?

7. How many are 12 from 14? 12 from 16? 12 from 15? 12 from 17? 12 from 19? 12 from 18? 12 from 20? 12 from 22? 12 from 23? 12 from 24?

8. Willie had 20 rabbits in the yard, and 12 of them got away; how many rabbits had he left?

WRITTEN EXERCISES.

Add:	23	27	30	34	34	36	37	39	47
	18	34	28	17	45	27	12	16	28
	24	28	31	35	33	35	38	49	68
	17	33	27	16	46	28	71	45	54
Subtract:	28	37	43	54	47	64	85	73	94
	16	13	25	36	25	26	48	28	67
	29	38	44	55	48	65	86	75	93
	17	14	26	37	26	27	49	37	48

LESSON XX.

Multiply and Divide with Eleven and Twelve.

1. How many are 2 times 11? 3 times 11? 4 times 11? 5 times 11? 6 times 11? 7 times 11? 8 times 11? 9 times 11? 10 times 11? 11 times 11? 12 times 11?

2. Complete the following and commit to memory:

$11 \times 2 = ?$ $11 \times 5 = ?$ $11 \times 8 = ?$ $11 \times 11 = ?$
$11 \times 3 = ?$ $11 \times 6 = ?$ $11 \times 9 = ?$ $11 \times 12 = ?$
$11 \times 4 = ?$ $11 \times 7 = ?$ $11 \times 10 =$

3. How far will a horse travel in 5 hours, at the rate of 11 miles an hour? In 8 hours? In 10 hours?

4. How many are 2 times 12? 3 times 12? 4 times 12? 5 times 12? 6 times 12? 7 times 12? 8 times 12? 9 times 12? 10 times 12? 11 times 12? 12 times 12?

5. Complete the following and commit to memory:

$12 \times 2 = ?$ $12 \times 5 = ?$ $12 \times 8 = ?$ $12 \times 11 = ?$
$12 \times 3 = ?$ $12 \times 6 = ?$ $12 \times 9 = ?$ $12 \times 12 = ?$
$12 \times 4 = ?$ $12 \times 7 = ?$ $12 \times 10 = ?$

6. How far will a steamboat go in 5 hours, at the rate of 12 miles an hour? In 7 hours? In 9 hours?

7. Write the Division Table for 11's. Write the Division Table for 12's.

WRITTEN EXERCISES.

Multiply:

34	41	39	45	73	86	72	34	35	46	57
6	5	3	4	4	5	9	11	11	11	11
24	40	38	46	74	87	74	53	34	47	56
7	8	3	5	6	7	8	12	12	12	12

Divide:

11)77	11)88	11)99	6)66	7)77	11)110	11)121	11)132
3)69	4)84	12)84	12)96	12)108	12)120	12)132	12)144

ORAL EXERCISES.

1. How many shoes are there in 3 pairs of shoes?

2. Nellie had half a dime and 2 cents; how many cents had she?

3. Ten is how many more than six? Five is how many less than eight?

4. If you had 12 cents, how many 2-cent stamps could you buy with the money?

5. If a pint of milk costs 3 cents, how much will a quart cost?

6. How many quart measures can you fill with 7 pints? How many pints will remain?

7. Divide 8 blocks into 2 equal groups; how many in each group? What part of 8 in each group?

8. If you had two 3-cent pieces, how many more cents would you require to have 10 cents?

9. If there are 6 boys and 5 girls in a class, how many pupils in the class?

10. Susie had half a dime and two 5-cent pieces; how many cents had she?

11. If Susie spent her money for peaches at 3 cents apiece, how many did she buy?

12. My father gave me a dime, one-half of which I spent for a ball; what did the ball cost me?

13. How many squares can you make with 12 sticks? How many triangles?

14. What is the greatest number of 3-cent pieces in 9 cents? In 11 cents?

15. Frank had 15 cents, and took a ride on a street-car, costing 5 cents; how much money remained?

16. Make a little story about 6 boys and 3 girls. About 8 cows and 4 horses.

17. If I have four 5-cent pieces, how many times can I ride in a street-car, the fare being 5 cents for each ride?

SIGHT EXERCISES.

1. Add:

3	6	3	4	3	5	6	7	6
4	5	6	8	6	7	4	3	5
7	4	2	7	8	3	7	8	4
2	6	9	8	5	6	3	4	7
4	7	4	2	7	7	2	7	3
8	3	7	5	6	5	9	8	6

2. Subtract:

9	8	12	13	18	14	16	18	15
5	6	7	4	6	7	9	8	6
15	16	19	18	20	20	19	17	20
8	7	12	10	12	9	8	13	15
19	20	20	12	20	20	20	20	18
11	8	17	15	16	11	18	12	15

3. Name products:

4×2	5×2	6×2	7×2	8×2
9×2	10×2	2×3	3×3	4×3
5×3	6×3	2×4	3×4	4×4
5×4	2×5	3×5	4×5	2×6
3×6	2×7	2×8	2×9	2×10

4. Name quotients:

$2 \div 2$	$4 \div 2$	$6 \div 2$	$8 \div 2$	$10 \div 2$
$12 \div 2$	$14 \div 2$	$16 \div 2$	$18 \div 2$	$20 \div 2$
$6 \div 3$	$9 \div 3$	$12 \div 3$	$15 \div 3$	$18 \div 3$
$8 \div 4$	$12 \div 4$	$16 \div 4$	$20 \div 4$	$10 \div 5$
$15 \div 5$	$20 \div 5$	$12 \div 6$	$18 \div 6$	$17 \div 7$
$14 \div 7$	$8 \div 8$	$16 \div 8$	$18 \div 9$	$20 \div 10$

ORAL EXERCISES.

1. A foot and 6 inches are how many inches? Half a foot and 10 inches are how many inches?

2. Florence has two 5-cent pieces; how many cents more must she have to make 16 cents?

3. What three pieces of money will make 15 cents? What two pieces of money will make 15 cents?

4. How many sides have 5 triangles? How many sides have 5 squares?

5. Three times what number equals 15? Five times what number equals 15?

6. Ella Smith is 15 years of age; how old was she 7 years ago?

7. There were 20 children in a class, but one stormy day only half were present; how many were absent?

8. How many triangles can you make with 17 sticks? How many sticks will be left?

9. How many squares can you make with 17 sticks? How many will be left?

10. Caroline has 3 half-dimes and two cents; how many cents has she?

11. There were 18 street-cars in a station; after 11 of them had gone out, how many were in the station?

12. A family ate a dozen and a half of eggs; how many eggs did they eat?

13. Willis is 8 years old to-day; in how many years will he be 15 years old?

14. Can you show what three pieces of money will make 15 cents? What three pieces will make 17 cents?

15. How many 5-cent stamps can you buy with 18 cents? How many cents will be left?

16. An organ-grinder received at one house 6 cents, at another 7 cents, and at another 5 cents; how many cents did he receive?

LESSON XXI.

Inexact Division.

1. Show with blocks or sticks how many 2's in 5; in 7; in 9; in 11; in 13.

2. Show with objects how many 3's in 5; in 7; in 10; in 13; in 19; in 20.

3. Show with objects how many 4's in 6; in 9; in 11; in 13; in 15; in 17; in 19.

4. Show, with the remainders, how many 3's in 5; in 8; in 11; in 14; in 16; in 19.

5. Show, with remainders, how many 4's in 7; in 10; in 14; in 17; in 18; in 21.

6. Show, with remainders, how many 5's in 6; in 8; in 12; in 16; in 18; in 22; in 24; in 27; in 33.

7. Find, with remainders, how many 6's in 7; in 10; in 14; in 16; in 21; in 25; in 28; in 31; in 34.

8. How many oranges can a boy get for 20 cents, at 3 cents apiece; and how many cents will remain?

9. Mary, having a quarter of a dollar, bought some pinks at 4 cents apiece; how many pinks did she get, and how many cents remained?

10. Edith, having half a dollar, bought some ribbon at 6 cents a yard; how many yards did she buy, and how many cents remained?

WRITTEN EXERCISES.

2)11	2)13	2)15	3)10	3)14	3)16	4)14	4)18
5)11	5)13	5)22	5)28	6)16	6)19	6)22	6)27
6)38	6)40	6)43	6)49	7)34	7)40	7)43	7)57
7)80	7)82	7)79	8)34	8)36	8)42	8)44	8)47
8)62	8)68	8)70	8)74	8)78	9)38	9)47	9)58
9)100	9)107	9)105	9)103	9)101	9)109	9)114	9)113

LESSON XXII.

Exercises in Inexact Division.

ORAL EXERCISES.

1. Find, with remainders, how many 7's in 9; in 11; in 15; in 18; in 22; in 25; in 30; in 34; in 36.

2. Find, with remainders, how many 8's in 10; in 12; in 14; in 17; in 20; in 26; in 30; in 34; in 36.

3. Find, with remainders, how many 9's in 11; in 13; in 16; in 19; in 21; in 26; in 32; in 35.

4. Mabel, having 75 cents, bought some roses at 8 cents apiece; how many did she get, and how much money remained?

5. Alice, having one dollar, bought some handkerchiefs at 12 cents each; how many did she buy, and what money remained?

WRITTEN EXERCISES.

1. Divide 75 by 3.

EXPLANATION.—Dividing the 7 tens by 3, we have a quotient of 2 tens and 1 ten remaining. This 1 ten and 5 units make 15 units. Dividing 15 units by 3, we have 5 units. Hence, dividing 75 by 3, we have 25.

OPERATION.

$$3\overline{)75}\quad 25$$

2. In the same manner pupils will make the following divisions:

3.	**4.**	**5.**	**6.**	**7.**	**8.**	**9.**
2)34	2)54	2)72	2)78	2)94	2)96	2)98
10.	**11.**	**12.**	**13.**	**14.**	**15.**	**16.**
3)42	3)48	3)54	3)57	3)78	3)81	3)84
17.	**18.**	**19.**	**20.**	**21.**	**22.**	**23.**
4)52	4)60	4)68	4)72	4)92	4)96	4)100
24.	**25.**	**26.**	**27.**	**28.**	**29.**	**30.**
5)60	5)70	5)80	5)85	5)95	5)100	5)115

31.	**32.**	**33.**	**34.**	**35.**	**36.**	**37.**
6)72	6)84	6)96	6)102	6)114	6)132	6)144
38.	**39.**	**40.**	**41.**	**42.**	**43.**	**44.**
7)84	7)98	7)112	7)119	7)133	7)154	7)168
45.	**46.**	**47.**	**48.**	**49.**	**50.**	**51.**
8)96	8)112	8)128	8)136	8)152	8)196	8)184
52.	**53.**	**54.**	**55.**	**56.**	**57.**	**58.**
9)108	9)126	9)144	9)153	9)171	9)198	9)207

SIGHT EXERCISES.

1. Add:

2	2	3	4	3	4	2	3	4	5
3	4	4	5	6	7	8	9	6	4
3	5	6	8	9	7	5	8	8	9
7	8	7	5	3	6	9	7	9	8
9	5	7	7	6	10	11	12	16	15
9	10	10	11	11	8	7	6	4	5

2. Subtract:

8	9	6	7	6	8	9	8	7	9
4	3	5	2	4	7	6	5	2	5
10	11	12	12	13	14	15	16	17	18
6	4	6	8	4	6	6	8	6	7
14	15	17	17	18	20	20	20	20	20
7	8	7	9	8	6	8	12	13	14

3. Find results:

$3+2+1$	$6+5-1$	$3+9-6$	$8+2-3$
$4+3+2$	$7+3-4$	$4+7-2$	$7-5+4$
$5+2+3$	$6+4-5$	$6+5-4$	$8-6+4$
$6+4+2$	$4+8-4$	$7-4+5$	$9-7+5$

LESSON XXIII.

Exercises with Larger Numbers.

1. How many are 12 and 2? 22 and 2? 32 and 2? 42 and 2? 52 and 2? 62 and 2? 72 and 2? 82 and 2?

2. How many are 14 minus 2? 24 minus 2? 34 minus 2? 44 minus 2? 54 minus 2? 64 minus 2? 74 minus 2?

3. How many are 22 and 3? 32 and 3? 42 and 3? 52 and 3? 62 and 3? 72 and 3? 82 and 3?

4. How many are 15 minus 3? 25 minus 3? 35 minus 3? 45 minus 3? 55 minus 3? 65 minus 3? 75 minus 3?

5. How many are 22 and 4? 32 and 4? 42 and 4? 52 and 4? 62 and 4? 72 and 4? 82 and 4?

6. How many are 16 minus 4? 26 minus 4? 36 minus 4? 46 minus 4? 56 minus 4? 66 minus 4? 76 minus 4? 86 minus 4?

7. How many are 23 and 5? 33 and 5? 43 and 5? 53 and 5? 63 and 5? 73 and 5? 83 and 5?

8. How many are 17 minus 5? 27 minus 5? 37 minus 5? 47 minus 5? 57 minus 5? 67 minus 5? 77 minus 5? 87 minus 5?

9. How many are 16 and 6? 26 and 6? 36 and 6? 46 and 6? 56 and 6? 66 and 6? 76 and 6?

10. How many are 18 minus 6? 28 minus 6? 38 minus 6? 48 minus 6? 58 minus 6? 68 minus 6? 78 minus 6?

11. How many are 14 and 8? 24 and 8? 34 and 8? 44 and 8? 54 and 8? 64 and 8? 74 and 8? 84 and 8?

12. How many are 99 minus 8? 89 minus 8? 79 minus 8? 69 minus 8? 59 minus 8? 49 minus 8? 39 minus 8? 29 minus 8?

13. How many are 3 and 13? 4 and 14? 5 and 15? 6 and 16? 7 and 17? 8 and 18?

ORAL EXERCISES IN REVIEW.

1. Eight pints of vinegar will fill how many quart cans?

2. A quart of milk costs 6 cents; what will a pint cost?

3. A quart of cream costs 8 cents; what will 5 pints cost?

4. If you have four 5-cent pieces and a dime, how many cents have you?

5. How many joints has each finger? How many joints have your four fingers on both hands?

6. John had a piece of string a foot long, and cut off 5 inches; how many inches were left?

7. Mabel had 3 pieces of money that made just 11 cents; what were the pieces?

8. Mabel gave her three pieces for two pieces making 11 cents; what were the two pieces?

9. I had 18 cents, and bought six 2-cent stamps; how much money had I left?

10. What two pieces of money can I use in paying 15 cents?

11. How many cents do you need to put with a dime and half a dime to make 21 cents?

12. I have 30 cents in 5-cent pieces; how many 5-cent pieces have I?

13. What two pieces of money will make 15 cents? What two pieces of money will make 30 cents?

14. Three times what number equals eighteen? Four times what number equals thirty-two?

15. How many oxen can be shod with 24 shoes if each ox requires 8 shoes?

16. How many dozen buttons has Alice if she has 36 buttons?

17. Willis lives 5 blocks from the school-house; how many blocks does he walk each day in attending school if he goes home to dinner?

WRITTEN EXERCISES IN REVIEW.

1. Willie weighs 56 pounds, and Annie weighs 48 pounds; how much do both weigh?

2. A newsboy sold papers for 97 cents, and gained 23 cents; how much did the papers cost him?

3. There are 45 roses on one bush, and 48 roses on another bush; how many roses on both bushes?

4. A grocer bought 124 eggs; how many would remain after selling 3 dozen eggs?

5. A lady bought 75 cents' worth of groceries, and gave the grocer a dollar bill; what was her change?

6. There are 6 rows of desks in a school-room, and 15 desks in each row; how many desks in the school-room?

7. If tea is worth 35 cents a half-pound, how much must a lady pay for a pound?

8. If there are 66 fire-crackers in a pack, how many are there in 8 packs?

9. A girl bought a doll for 35 cents; she received 65 cents change; how much money did she hand the store-keeper?

10. Frances paid 85 cents for 5 pounds of candy; what did the candy cost a pound?

11. A street-car conductor collected 90 cents in one trip; how many people paid at 5 cents a fare?

12. Two boys together weigh 105 pounds, and one boy weighs 68 pounds; what does the other boy weigh?

13. Elsie went to school 5 months, going 21 days each month; how many days did she attend school?

14. Required the cost of 5 pounds of 60-cent tea and a pound of 38-cent coffee.

15. At 5 cents a pound, how many pounds of oatmeal can I get for 125 cents?

16. A man bought a horse for 105 dollars; how many 5-dollar bills will it take to pay for the horse?

LESSON XXIV.

The Multiplication Table.

1. We will now collect the elementary products into a table called the Multiplication Table.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

EXPLANATION.—The numbers at the top may be multiplied successively by the numbers in the column at the left, the products being found in the vertical column below the number multiplied.

Thus, two 2's are 4; three 2's are 6; four 2's are 8, etc. This order may be reversed at the option of the teacher, reading "two 2's are 4, two 3's are 6," etc.

NOTES.—1. The shading indicates the products most difficult to remember. Teachers should give special attention to these products.

2. The table is complete for practical purposes at "9 times 9," but custom has extended it to "12 times 12."

LESSON XXV.

The Roman Notation.

1. We sometimes express numbers by the use of capital letters, as follows:

I = 1; V = 5; X = 10; L = 50; C = 100;
D = 500; M = 1000.

2. To express other numbers these letters are written

beside one another. The first twenty numbers are expressed as follows:

I = 1.	VI = 6.	XI = 11.	XVI = 16.
II = 2.	VII = 7.	XII = 12.	XVII = 17.
III = 3.	VIII = 8.	XIII = 13.	XVIII = 18.
IV = 4.	IX = 9.	XIV = 14.	XIX = 19.
V = 5.	X = 10.	XV = 15.	XX = 20.

TIME BY THE CLOCK.

1. Point out the figures on the face of the clock.

2. What time is it by the clock?

3. What difference do you see in the hands?

4. Which hand points to III? Since the shorter hand points out the hours, what may we call it?

Ans. The *hour hand.*

5. Which hand points to XII? Since the longer hand points out the minutes, what may we call it? *Ans.* The *minute hand.*

6. How many little spaces from XII to I? From XII to II? What are these spaces called? *Ans. Minute spaces.*

7. How many minute spaces in the whole circle? How long does it take the minute hand to go around the circle?

8. How many minutes, then, in one hour?

9. How many minutes in half an hour? In a quarter of an hour? In 2 hours?

10. Show where the two hands will be at 2 o'clock. At 3 o'clock. At 4 o'clock. At 6 o'clock. At 9 o'clock.

11. Show where the two hands will be at quarter after 3 o'clock. At half after 6 o'clock. At quarter of 8 o'clock.

12. How long does it take the hour hand to go around the circle? What part of a day is that? How many hours in half a day? How many hours in a day?

SECTION IV.

INTRODUCTION TO FRACTIONS.

LESSON I.

One-half.

1. If I divide an apple into *two equal parts*, what is *one* of the parts called? *Ans. One-half.*

2. How many *halves* of an apple in *one* apple?

3. If I divide anything into *two equal parts*, what is each part called?

4. What do we understand by *one-half* of anything?

Ans. One-half is one of the two equal parts into which anything is divided.

5. What is one-half of 6?

SOLUTION.—One-half of 6 is 3, since two 3's make 6.

6. What is 1 half of 4? Of 8? Of 10? Of 12?

7. What is 1 half of 16? Of 14? Of 20? Of 18?

8. What is 1 half of 24? Of 22? Of 26? Of 28?

9. If a yard of tape costs 4 cents, what will 1 half of a yard cost?

SOLUTION.—If 1 yard of tape costs 4 cents, 1 half of a yard will cost 1 half of 4 cents, which is 2 cents.

10. Edith had 8 oranges, and gave 1 half of them to her brother; how many did she give away?

11. Mary had 16 apples, and gave 1 half of them to Sarah; how many apples did Sarah receive?

WRITTEN EXERCISES.

1. *One-half* is written thus: $\frac{1}{2}$: *two-halves*, $\frac{2}{2}$.

2. What is $\frac{1}{2}$ of 24? $\frac{1}{2}$ of 46? $\frac{1}{2}$ of 84? $\frac{1}{2}$ of 102?

3. Find $\frac{1}{2}$ of 48; $\frac{1}{2}$ of 96; $\frac{1}{2}$ of 274; $\frac{1}{2}$ of 452.

4. A farmer raised 80 bushels of apples, and sold $\frac{1}{2}$ of them; how many did he sell?

LESSON II.

One-third.

1. If I divide an apple into *three equal parts*, what is *one* of the parts called? *Ans. One-third.*

2. What are *two* parts called? *Three* parts? How many *thirds* in *one* apple?

3. What is *one-third? Ans. One-third is one of the three equal parts into which anything is divided.*

4. What is one-third of 6? *Ans. One-third of 6 is 2, since three 2's make 6.*

5. What is 1 third of 9? Of 12? Of 18? Of 15?

6. What is 1 third of 24? Of 30? Of 27? Of 33?

7. Willis had 15 apples, and gave 1 third of them to Ada; how many did he give to Ada?

SOLUTION.—If Willis had 15 apples, and gave 1 third of them to Ada, he gave to Ada 1 third of 15 apples, or 5 apples.

8. Ada had 18 plums, and gave 1 third of them to Edith; how many plums did she give to Edith?

9. What are two-thirds of 6?

SOLUTION.—One-third of 6 is 2, and two-thirds of 6 are 2 times 2, which are 4.

10. What are 2 thirds of 9? Of 12? Of 15? Of 18?

11. What are 2 thirds of 24? Of 21? Of 30? Of 36?

12. Laura bought 9 lemons, and gave Effie 2 thirds of them; how many did she give to Effie?

WRITTEN EXERCISES.

1. The fractions *one-third*, *two-thirds*, etc. are written thus:

One-third,	*Two-thirds,*	*Three-thirds,*	*Four-thirds.*
$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{4}{3}$

2. What is $\frac{1}{3}$ of 48? $\frac{1}{3}$ of 54? $\frac{1}{3}$ of 72? $\frac{1}{3}$ of 75?

3. Find $\frac{2}{3}$ of 60; $\frac{2}{3}$ of 84; $\frac{2}{3}$ of 216; $\frac{2}{3}$ of 237.

4. Mr. Wilson earned 180 dollars, and spent $\frac{1}{3}$ of it; how much did he spend? How much remained?

LESSON III.

One-fourth.

1. If I divide a square into four equal parts, what is one of these parts called?

2. What are 2 parts called? 3 parts? 4 parts?

3. How many fourths of a square in one square?

4. What is 1 fourth of 8?

SOLUTION.—One-fourth of 8 is 2, since four 2's are 8.

5. What is 1 fourth of 12? Of 16? Of 24? Of 36?

6. What is 1 fourth of 20? Of 28? Of 32? Of 44?

7. A farmer having 12 pigs sold 1 fourth of them to his neighbor; how many pigs did he sell?

8. If Henry gave his sister 1 fourth of 24 pears, how many pears did his sister receive?

9. What are 2 fourths of 16?

SOLUTION.—One-fourth of 16 is 4, and 2 fourths of 16 are 2 times 4, which are 8.

10. What are 2 fourths of 12? Of 24? Of 20? Of 32?

11. What are 3 fourths of 20? Of 16? Of 36? Of 44?

12. Mason gave his sister 2 fourths of 12 oranges; how many did she receive?

13. Henry has 32 cents, and his brother has 3 fourths as many; how many cents have both?

WRITTEN EXERCISES.

1. The fractions *one-fourth, two-fourths,* etc., are written as follows:

One-fourth,	***Two-fourths,***	***Three-fourths,***	***Four-fourths.***
$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{4}{4}$

2. What is $\frac{1}{4}$ of 64? Of 96? Of 100? Of 132?

3. What are $\frac{2}{4}$ of 48? Of 80? Of 136? Of 236?

4. What are $\frac{3}{4}$ of 84? Of 96? Of 216? Of 368?

5. If 4 acres of meadow-land are worth 480 dollars, what is one acre worth?

LESSON IV.

One-fifth.

1. If we divide a line into five equal parts, what is one of the parts called?

2. What are 2 parts called? 3 parts? 4 parts? 5 parts? How many fifths in anything?

3. What is one-fifth of 10? *Ans. One-fifth of 10 is 2, since five 2's are 10.*

4. What is 1 fifth of 15? Of 20? Of 35? Of 40?

5. What are 2 fifths of 15? Of 30? Of 45? Of 50?

6. What are 3 fifths of 20? Of 25? Of 55? Of 45?

7. What are 4 fifths of 25? Of 35? Of 40? Of 60?

8. Rachel has 20 oranges, and Mary has 1 fifth as many; how many has Mary?

9. Fanny's book cost 30 cents, and Frank's book cost 3 fifths as much; what was the cost of Frank's book?

10. A cow cost 40 dollars, and a colt cost 4 fifths as much; required the cost of the colt.

11. Jansen bought 3 fourths of 40 pears, and sold 10 of them; how many remained?

12. Edith has 1 third of 30 roses, and Florence has 1 half of 20 roses; how many have they both?

WRITTEN EXERCISES.

1. The fractions *one-fifth*, *two-fifths*, etc. are written as follows:

One-fifth,	*Two-fifths,*	*Three-fifths,*	*Five-fifths.*
$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{5}{5}$

2. Required the value of the following:

$\frac{1}{5}$ of 75; $\frac{1}{5}$ of 105; $\frac{2}{5}$ of 135; $\frac{2}{5}$ of 230; $\frac{3}{5}$ of 240;
$\frac{2}{5}$ of 95; $\frac{3}{5}$ of 360; $\frac{4}{5}$ of 445; $\frac{3}{5}$ of 650; $\frac{4}{5}$ of 565.

3. If 5 cows cost a drover 320 dollars, how much did he pay for each cow?

LESSON V.

One-sixth and One-seventh.

1. If you divide an oblong into six equal parts, what is 1 part called? 2 parts? 3 parts? etc.

2. How many sixths in *one?* What is one-sixth of anything?

3. What is 1 sixth of 18? Of 12? Of 24? Of 30?

4. What are 2 sixths of 12? Of 36? Of 42? Of 48?

5. What are 3 sixths of 36? Of 42? Of 60? Of 66?

6. What are 5 sixths of 42? Of 48? Of 60? Of 72?

7. Josie had 24 plums, and gave 2 sixths of them away; how many did she give away?

8. Arthur bought 30 pens, and sold 3 sixths of them; how many did he sell?

9. Joseph walked 36 miles, and rode 5 sixths as far; how far did he ride?

10. If you divide a line into 7 equal parts, what is 1 of these parts called? 2 parts? 3 parts? etc.

11. How many sevenths in one? What is one-seventh of anything?

12. What is 1 seventh of 14? Of 21? Of 35? Of 42?

13. What are 2 sevenths of 7? Of 28? Of 42? Of 77?

14. What are 3 sevenths of 21? Of 70? Of 49? Of 35? Of 63?

15. What are 4 sevenths of 28? Of 35? Of 56? Of 49? Of 70?

16. What are 5 sevenths of 42? Of 63? Of 49? Of 28? Of 84?

17. A fish-line is 21 feet long, and the pole is 6 sevenths as long; how long is the pole?

18. Eliza had 42 roses, and Lizzie had 4 sevenths as many as Eliza; how many roses had Lizzie?

19. Martin walked 28 miles one day, and 6 sevenths as far the next day; how far did he walk both days?

WRITTEN EXERCISES.

1. *One-sixth* is written $\frac{1}{6}$; 2 *sixths*, $\frac{2}{6}$; 3 *sixths*, $\frac{3}{6}$; 1 *seventh*, $\frac{1}{7}$; 2 *sevenths*, $\frac{2}{7}$, etc.

2. Required the value of the following:

$\frac{1}{6}$ of 84; $\frac{1}{6}$ of 108; $\frac{2}{6}$ of 216; $\frac{3}{6}$ of 570; $\frac{4}{6}$ of 720; $\frac{5}{6}$ of 840; $\frac{1}{7}$ of 98; $\frac{1}{7}$ of 203; $\frac{2}{7}$ of 287; $\frac{3}{7}$ of 679; $\frac{4}{7}$ of 896; $\frac{6}{7}$ of 952.

3. A farmer raised 504 bushels of oats, and sold $\frac{5}{6}$ of the crop; how many bushels did he sell?

4. If a locomotive runs 840 miles in a week, how far did it run in $\frac{6}{7}$ of a week?

LESSON VI.

One-eighth, One-ninth, One-tenth, etc.

1. If a square is divided into 8 equal parts, what is 1 part called? 2 parts? 3 parts? etc.

2. How many eighths in 1? What is one-eighth of anything?

3. What is 1 eighth of 16? Of 32? Of 24? Of 48?

4. What are 3 eighths of 40? Of 56? Of 64? Of 72?

5. What are 5 eighths of 48? Of 80? Of 88? Of 96?

6. A watch was bought for 24 dollars, and sold for 5 eighths of its cost; for what was it sold?

7. Norris caught 40 fish, and sold 4 eighths of them; how many fish did he keep?

8. Frank ran 48 rods, and John ran 5 eighths as far; how much farther did Frank run than John?

9. If an oblong is divided into nine equal parts, what is each part called? 2 parts? 3 parts? etc.

10. How many ninths in one? What is one-ninth of anything?

11. What are 2 ninths of 18? Of 36? Of 27? Of 63?

12. What are 4 ninths of 36? Of 72? Of 63? Of 81?

13. What are 7 ninths of 18? Of 27? Of 72? Of 99?

14. What are 8 ninths of 27? Of 36? Of 72? Of 108?

15. If a yard of flannel costs 72 cents, how much will 5 ninths of a yard cost?

16. Andrew ran 36 rods, and Amos ran 4 ninths as far; how far did Amos run?

17. A farmer raised 72 sheep, and sold 8 ninths of them; how many did he retain?

18. What is 1 tenth of anything? What is 1 eleventh of anything? What is 1 twelfth of anything?

19. What is 1 tenth of 30? 2 tenths of 40? 3 tenths of 60? 4 tenths of 80?

20. What is 1 eleventh of 33? 3 elevenths of 44? 6 elevenths of 66? 8 elevenths of 88?

21. What is 1 twelfth of 24? 3 twelfths of 36? 5 twelfths of 84? 7 twelfths of 108?

22. A drover, having 60 horses, sold 5 tenths of them; how many then remained?

23. How much will 5 elevenths of 55 yards of cloth cost, at 5 dollars a yard?

WRITTEN EXERCISES.

1. *One-eighth* is written $\frac{1}{8}$; 2 *eighths*, $\frac{2}{8}$; 3 *eighths*, $\frac{3}{8}$; 1 *ninth*, $\frac{1}{9}$; 2 *ninths*, $\frac{2}{9}$, etc.

2. Required the value of the following:

$\frac{1}{8}$ of 88; $\frac{2}{8}$ of 96; $\frac{3}{8}$ of 160; $\frac{4}{8}$ of 176; $\frac{5}{8}$ of 272;
$\frac{1}{9}$ of 99; $\frac{2}{9}$ of 108; $\frac{4}{9}$ of 117; $\frac{5}{9}$ of 126; $\frac{8}{9}$ of 144.

3. *One-tenth* is written $\frac{1}{10}$; 2 *tenths*, $\frac{2}{10}$; *one-eleventh*, $\frac{1}{11}$; 2 *elevenths*, $\frac{2}{11}$; *one-twelfth*, $\frac{1}{12}$; etc.

4. Find $\frac{1}{10}$ of 250; $\frac{2}{10}$ of 360; $\frac{7}{10}$ of 560; $\frac{1}{11}$ of 660; $\frac{5}{11}$ of 341; $\frac{3}{12}$ of 240; $\frac{11}{12}$ of 360.

5. If an engine runs 320 miles in 8 hours, how far will it run in one hour? How far will it run in 6 hours?

SECTION V.

FUNDAMENTAL OPERATIONS.

NUMERATION AND NOTATION.

1. In writing numbers we use the following ten characters, called *figures:*

1,	2,	3,	4,	5,	6,	7,	8,	9,	0.
One,	Two,	Three,	Four,	Five,	Six,	Seven,	Eight,	Nine,	Naught.

2. These figures are combined to express larger numbers.

3. In a number expressed by two figures, the figure on the right expresses *ones*, or *units*, and the figure on the left expresses *tens*.

Thus, in 46 the 6 expresses 6 *units* and the 4 expresses 4 *tens*, and the number is read *forty-six*.

4. In a number expressed by three figures, the third figure toward the left expresses *hundreds*.

Thus, in 345 the 3 expresses 3 *hundreds*, the 4 expresses 4 *tens*, etc., and the number is read *three hundred forty-five*.

The pupil will read the following:

1. 23.	**4.** 348.	**7.** 572.	**10.** 682.	**13.** 658.
2. 24.	**5.** 455.	**8.** 407.	**11.** 574.	**14.** 760.
3. 353.	**6.** 564.	**9.** 669.	**12.** 707.	**15.** 870.

The pupil will write the following in figures:

1. One hundred forty-five.
2. Two hundred sixty-seven.
3. Three hundred fifty-eight.
4. Ninety-nine.
5. Four hundred forty-four.
6. Five hundred sixty.
7. Six hundred thirty-two.
8. Seven hundred forty-six.
9. Eight hundred sixty-four.
10. Nine hundred fifty-nine.

5. When four figures are combined to express an integer, the fourth figure toward the left expresses *thousands*.

Thus, in 4678 the 4 expresses 4 *thousands*, the 6 expresses 6 *hundreds*, etc.

The pupil will read the following:

1. 3456.	**4.** 4273.	**7.** 5276.	**10.** 7368.
2. 2547.	**5.** 5168.	**8.** 6378.	**11.** 8572.
3. 3675.	**6.** 4735.	**9.** 5839.	**12.** 9467.

The pupil will write the following in figures:

1. Two thousand three hundred twenty-four.

2. Three thousand five hundred nineteen.

3. Four thousand six hundred thirty-five.

4. One thousand eight hundred seven.

5. Five thousand three hundred sixty-one.

6. Six thousand eight hundred forty-two.

7. Seven thousand four hundred seventy-seven.

8. Eight thousand forty-four.

6. The **Method** of expressing numbers by figures is as follows:

1. *A figure standing alone expresses* ONES, or UNITS.

2. *A figure in the second place expresses* TENS.

3. *A figure in the third place expresses* HUNDREDS; *in the fourth place,* THOUSANDS, *etc.*

7. The following little table shows the names of the first twelve places:

NUMERATION TABLE.

NAMES.	Hundred-billions.	Ten-billions.	Billions.	Hundred-millions.	Ten-millions.	Millions.	Hundred-thousands.	Ten thousands.	Thousands.	Hundreds.	Tens.	Units.
	5	5	5	3	5	5	5	7	3	2	6	4
PLACES.	12th,	11th,	10th,	9th,	8th,	7th,	6th,	5th,	4th,	3d,	2d,	1st,
PERIODS.	4th.			3d.			2d.			1st.		

8. The first three places are called *units period.* Thus, the 264 expresses *two hundred sixty-four* UNITS.

9. The second three places are called *thousands period.* Thus the 573 expresses *five hundred seventy-three* THOUSANDS.

10. The third three places are called *millions period.* The fourth three places are called *billions period.*

NOTE.—The teacher will explain this until it is clearly understood.

ORAL EXERCISES.

1. The pupil will give the name of each of the following places:

1. First.	**4.** Fourth.	**7.** Seventh.
2. Third.	**5.** Sixth.	**8.** Eighth.
3. Second.	**6.** Fifth.	**9.** Ninth.

2. The pupil will give the number of the place of each of the following:

1. Units.	**4.** Thousands.	**7.** Ten-thousands.
2. Tens.	**5.** Millions.	**8.** Ten-millions.
3. Hundreds.	**6.** Billions.	**9.** Hundred-thousands.

11. The method of reading numbers expressed by figures is called **Numeration.**

EXERCISES IN NUMERATION.

1. Write and read 4356568.

SOLUTION.—We separate the expression into periods of three figures each. The third period is 4 *million*, the second period is 356 *thousand*, the first period is 568; and the number is read 4 million, 356 thousand, five hundred sixty-eight.

OPERATION.
4,356,568.

We thus read numbers by the following method:

I. *Begin at the right hand, and separate the numerical expression into periods of three figures each.*

II. *Then begin at the left hand, and read each period as if it stood alone, giving the name of each period except the last.*

Read the following:

2. 2567	**6.** 25000	**10.** 34675
3. 4635	**7.** 28406	**11.** 380000
4. 7284	**8.** 30094	**12.** 2567534
5. 8506	**9.** 268750	**13.** 46785632

12. The method of writing numbers is called **Notation.**

EXERCISES IN NOTATION.

1. Write the number *five thousand three hundred six.*

SOLUTION.—We write the 5 thousand in the 4th place, the 3 hundred in the 3d place, a cipher in the 2d place, and the 6 units in the 1st place.

OPERATION.
5,306

We thus write numbers by the following method:

I. *Begin at the left hand and write the hundreds, tens, and units of each period in their proper order.*

II. *When there are vacant places fill them with ciphers.*

Write the following numbers:

1. One hundred six.

2. Four hundred ten.

3. Five hundred forty.

4. One thousand, sixty.

5. Six thousand, four hundred eighty-five.

6. Six thousand, eighty-five.

7. Seven thousand, sixty-four.

8. Forty-seven thousand, six hundred fifty.

9. One hundred fifty thousand, sixty-eight.

10. One million, four hundred sixty-five thousand, six hundred eight.

11. Four million, three hundred seven thousand, forty.

12. Fifteen million, four hundred four thousand, seventy-four.

13. Forty-eight million, six hundred eighty-five thousand, three hundred seven.

14. Seventy-nine million, fifty-four thousand, six hundred two.

15. Three hundred million, fifty-five hundred twenty-eight.

16. Three billion, three hundred four million, twenty-five thousand, six.

17. What number comes just before 200? 500? 700? 900? 1000? 10,000?

18. What is the smallest number you can write with three figures? With four figures?

19. What is the largest number you can write with three figures? With four figures?

20. What is the smallest number you can write with five figures? With six figures? With seven figures?

NOTE.—The teacher will drill the pupil in such exercises until he can read and write numbers readily. Do not expect to complete everything at first.

NOTATION OF UNITED STATES MONEY.

13. The money of the United States consists of *dollars* and *cents*. One dollar is equal to 100 cents.

14. The **Sign for dollars** is \$. It is written before the number of dollars. Thus, \$25 means 25 dollars.

15. In writing dollars and cents a period, called the *decimal* point, is placed between dollars and cents. Thus, \$6.45 is read 6 dollars 45 cents.

16. When there are no dollars, we write a zero, 0, in the place of dollars. Thus 75 cents is written \$0.75.

17. When the number of cents is less than 10, we write a cipher between the decimal point and the number of cents. Thus, 5 cents is written \$0.05.

NUMERATION.

Read the following:

1. \$6.75.	**3.** \$12.35.	**5.** \$25.50.	**7.** \$15.05.
2. \$8.25.	**4.** \$15.65.	**6.** \$36.85.	**8.** \$75.08.

NOTATION.

Write the following:

1. Six dollars four cents.
2. Ten dollars six cents.
3. Five dollars ten cents.
4. Six dollars five cents.

5. Seven dollars sixty-eight cents.

6. Nine dollars eighty-seven cents.

7. Twelve dollars eighteen cents.

8. Twenty-five dollars twenty-five cents.

9. Fourteen dollars five cents.

10. Eighteen dollars eight cents.

11. Twenty-four dollars fourteen cents.

12. One hundred twenty-five dollars eight cents.

13. Six hundred forty-five dollars thirty-eight cents.

14. Eight hundred fifty dollars seventy-five cents.

ROMAN NOTATION.

18. The **Roman Method** of Notation employs seven letters of the Roman Alphabet. Thus,

LETTERS,	I,	V,	X,	L,	C,	D,	M,
VALUES,	1,	5,	10,	50,	100,	500,	1000.

19. To express other numbers these characters are combined according to the following principles:

1. *Every time a letter is repeated its value is repeated.*

Thus, X denotes ten; XX, twenty; XXX, thirty; CC, two hundred, etc.

2. *When a letter is placed before one of greater value, the* DIFFERENCE *of their values is the number represented.*

Thus, I denotes one, V five, but IV denotes four. So also X denotes ten, but IX denotes nine.

3. *When a letter is placed after one of a greater value, the* SUM *of their values is the number represented.*

Thus, XV denotes fifteen; LX, sixty; LXX, seventy; MC, one thousand one hundred.

4. *When a letter is placed between two letters each of greater value than itself, its value is to be taken from the sum of the value of the other two.*

Thus, XIV denotes fourteen; XIX, nineteen; CIX, one hundred nine.

5. *A dash placed over a letter increases its value a thousand-fold.*

Thus, $\overline{V}$ denotes five thousand; $\overline{VII}$, seven thousand; $\overline{LX}$, sixty thousand.

ROMAN TABLE.

I . . .	One.	XXX .	Thirty.
II . . .	Two.	XL .	Forty.
III . . .	Three.	L .	Fifty.
IV . . .	Four.	LX .	Sixty.
V . . .	Five.	LXX .	Seventy.
VI . . .	Six.	XC .	Ninety.
VII . . .	Seven.	C .	One hundred.
VIII . . .	Eight.	CC .	Two hundred.
IX . . .	Nine.	D .	Five hundred.
X . . .	Ten.	DC .	Six hundred.
XI . . .	Eleven.	DCCCC .	Nine hundred.
XIV . . .	Fourteen.	M .	One thousand.
XV . . .	Fifteen.	MM .	Two thousand.
XIX . . .	Nineteen.	MCLX .	One thousand one hun-
XX . . .	Twenty.	MDCCCXCV 1895	[dred and sixty.

NUMERATION.

Read the following:

1. XXI.	**4.** XXVI.	**7.** XLI.	**10.** XCIX.
2. XXV.	**5.** XXXV.	**8.** XLIX.	**11.** CXLII.
3. XXIV.	**6.** XXXVI.	**9.** LVIII.	**12.** DCLXV.

Express the following by the Roman Notation:

1. 24.	**4.** 38.	**7.** 87.	**10.** 206.	**13.** 1052.
2. 37.	**5.** 69.	**8.** 78.	**11.** 356.	**14.** 1470.
3. 46.	**6.** 48.	**9.** 96.	**12.** 578.	**15.** 8000.

CHANGING DENOMINATIONS.

1. How many cents in $2.45?
2. How many cents in $15.50?
3. How many dollars in 1850 cents?
4. How many cents in $25.75?
5. How many cents in $48.05?
6. How many dollars in 2005 cents?

ADDITION.

20. The process of finding the *sum* of two or more numbers is called **Addition.**

21. The **sum** of two or more numbers is a number that contains as many units as those numbers.

22. The **Sign of Addition** is +, and is read *plus.* It is placed between the numbers to be added.

23. The **Sign of Equality** is =, and is read *equals* or *is equal to.* Thus, $4 + 5 = 9$ is "4 plus 5 equals 9."

24. To add when the sum of no column exceeds nine units of that column.

1. Find the sum of 32, 24, and 42.

SOLUTION.—We write the numbers so that units are under units and tens under tens, draw a line beneath, and begin at the right to add. 2 and 4 are 6, and 2 are 8; we write the 8 in units place: 3 and 2 are 5, and 4 are 9; we write the 9 in tens place, and we have 98.

OPERATION.

$$\begin{array}{r} 42 \\ 24 \\ 32 \\ \hline 98 \end{array}$$

WRITTEN EXERCISES.

Find the sum of

2. 30, 23, and 24.
3. 21, 22, and 35.
4. 34, 21, and 43.
5. 121, 202, and 543.
6. 153, 214, and 312.
7. 610, 156, and 213.
8. 432, 234, and 321.
9. 216, 501, and 172.
10. 413, 212, and 164.
11. 731, 146, and 102.
12. 182, 503, and 304.
13. 1632, 4153, and 1202.
14. 1412, 1231, and 4325.
15. 2715, 4130, and 3054.
16. 2131, 1234, and 3122.
17. 1207, 3040, and 2430.
18. 2051, 3027, and 1500.
19. 12201, 23021, and 2142.
20. 50273, 17202, and 21310.
21. 21021, 2712, and 12032.
22. 1212, 31402, 30321, 17032, and 20031.

ORAL EXERCISES.

1. If I see 8 birds on one tree and 6 birds on another tree, how many birds do I see?

SOLUTION.—If I see 8 birds on one tree and 6 birds on another tree, I see 8 birds plus 6 birds, which are 14 birds.

2. Florence wrote 8 lines one day, and 12 lines the next day; how many lines did she write in both days?

3. Mamie has a bouquet of 12 roses, and Susie has one of 10 roses; how many roses in both bouquets?

4. If there are 15 boys and 12 girls in a class, how many scholars are there in the class?

5. I saw 15 crows on a tree, and 13 crows in the field; how many crows did I see in all?

6. Begin at 2 and add by 2's as far as 24. Begin at 1 and add by 2's as far as 25.

7. Begin at 3 and add by 3's to 30. Begin at 1 and add by 3's to 31. Begin at 2 and add by 2's to 32.

8. What is the value of $7 + 3 + 4$? Of $8 + 4 + 5$? Of $5 + 7 + 6$? Of $5 + 6 + 8$? Of $6 + 7 + 8$? Of $7 + 8 + 9$?

WRITTEN EXERCISES.

1. A man gave his son \$205 and his daughter \$670; how much did he give to both?

2. John takes 472 steps in going to school, and George takes 517 steps; how many steps do both take?

3. A horse cost \$145, a pair of oxen \$120, and a cow \$30; what did they all cost?

4. A newsboy sold 423 papers one week and 365 papers the next week; how many papers did he sell in both weeks?

5. There are 325 boys and 473 girls in a school; how many pupils in the school?

6. A boy counted 342 stars and a girl counted 425 stars; how many stars did they both count?

7. A drover sold 328 cattle, and had 460 left; how many had he before the sale?

8. A merchant sold a lot of goods for $352, and lost $136; how much did the goods cost him?

9. A farmer raised 3160 bushels of wheat, 507 bushels of oats, and 2322 bushels of corn; how many bushels of grain did he raise?

10. A man saved $2135 one year, $1502 the next year, and $5061 the third year; how much did he save in three years?

25. To add when the sum of a column exceeds nine units of that column.

1. What is the sum of 48, 27, and 93.

SOLUTION.—We write the numbers with units under units and tens under tens, draw a line beneath, and begin at the right to add. 8 and 7 are 15, and 3 are 18 units; we write the 8 units, and add the 1 ten to the tens column: 1 and 4 are 5, and 2 are 7, and 9 are 16 tens; we write the 16 tens; hence the sum is 168.

OPERATION.

$$\begin{array}{r} 93 \\ 27 \\ 48 \\ \hline 168 \end{array}$$

1. Lead the pupil to add by naming the partial results without naming the terms added. Thus, in the example given have them say, 8, 15, 18; 5, 7, 16.

2. The process of adding the tens of a sum to the next column is called *Carrying*.

WRITTEN EXERCISES.

2. Add 35, 42, and 67.
3. Add 28, 43, and 54.
4. Add 36, 75, and 45.
5. Add 29, 32, and 54.
6. Add 56, 73, and 48.
7. Add 65, 39, and 75.
8. Add 135, 372, and 460.
9. Add 346, 542, and 708.
10. Add 209, 843, and 325.
11. Add 725, 634, and 580.
12. Add 576, 305, and 617.
13. Add 813, 794, and 241.

(14)	(15)	(16)	(17)	(18)	(19)
927	745	242	318	224	703
835	312	715	619	846	681
212	764	343	230	715	185
130	123	525	541	204	518

(20)	(21)	(22)	(23)	(24)	(25)
361	712	876	25	2028	5607
204	172	91	386	5634	4073
328	845	127	42	2040	5430
506	819	50	407	4907	785
284	271	123	53	2151	35

(26)	(27)	(28)	(29)	(30)	(31)
4215	2051	5136	1072	5072	7121
7563	3627	1527	3905	1523	2018
3142	4135	380	273	4141	562
2565	2740	215	1512	203	43
3013	3165	51	46	3725	5607

ORAL EXERCISES.

1. If there are 5 birds on one tree, 6 birds on another tree, and 8 birds on another tree, how many birds are there in all?

SOLUTION.—In all there are 5 birds plus 6 birds, which are 11 birds, plus 8 birds, which are 19 birds. Therefore, etc.

2. Two birds fly in opposite directions, one 8 miles and the other 13 miles; how far are they apart?

3. Eddie has 12 cents, and his sister has 18 cents more than he has; how many cents have they both?

4. Alice spelled 15 words and missed 12 words; how many words did she try to spell?

5. Eunice solved 16 examples one week and 14 the next week; how many examples did she solve in the two weeks?

6. If I catch 5 fish one day, 6 the next day, and 7 the next day, how many fish do I catch in the three days?

7. In one hen's nest there are 7 eggs, in another 5 eggs, and in another 8 eggs; how many eggs are in the three nests?

8. If I have 12 cents, and my mother gives me 10 cents, and my father gives me 8 cents, how many cents shall I then have?

9. A girl paid 12 cents for a slate, 6 cents for a pencil-holder, and 5 cents for candies; how much did she spend for all?

10. Begin at 4 and add by 4's as far as 32. Begin at 1 and add by 4's as far as 33. Begin at 2 and add by 4's as 34. Begin at 3 and add by 4's as far as 35.

WRITTEN EXERCISES.

1. Mary's doll cost 85 cents, and a little cradle for it cost 135 cents; how much did both cost?

SOLUTION.—If Mary's doll cost 85 cents, and a little cradle cost 135 cents, they both cost the sum of 85 cents and 135 cents, which is 220 cents.

OPERATION.

$$\begin{array}{r} 85 \\ 135 \\ \hline 220 \end{array}$$

2. Two boats sail in opposite directions, one 84 miles, the other 78 miles; how far are they apart?

3. May solved 15 examples Monday, 16 Tuesday, and 18 Wednesday; how many examples did she solve in the three days?

4. James took 578 steps in going to school, and Jennie took 657 steps; how many steps did they both take?

5. John weighs 78 pounds, and Henry weighs 12 pounds more; how much do they both weigh?

6. A farmer sold 18 turkeys to one grocer, 23 to another, and 35 to another; how many did he sell in all?

7. A horse travelled 46 miles in one day, and 14 miles further the next day; how far did he travel in both days?

8. If I pay $4.50 for a pair of shoes and $3.75 for a hat, what do I pay for both?

9. Begin with 25, and find the sum of 25 and all the numbers between it and 32.

10. Willie had 355 cents in his money-box, and his grandmother gave him 187 cents; how many dollars and cents had he then?

11. My slate cost 25 cents and my reader 56 cents, my arithmetic 68 cents and my geography $1.25; what was the cost of all?

12. A florist sold 38 roses, 47 pinks, and 28 lilies; how many flowers did he sell?

13. Fannie spelled 64 words one week, 76 the next week, and 85 the next week; how many words did she spell in the three weeks?

14. Willie's top cost 45 cents, his whip cost 57 cents, his ball 75 cents, and his bat 35 cents; what was the cost of them all?

15. A drover sold 486 cows one year, 2560 the next year, and 4765 the following year; how many cows did he sell in the three years?

16. Sarah wrote thirty-eight words one week, seventy-five the next week, and ninety-six the third week; how many words did she write in all?

17. I had eighty cents in my money-bank; my father put in ninety-five cents, and my mother put in eighty-seven cents; how much was then in the bank?

18. A farmer raised three hundred five bushels of wheat, seven hundred forty-seven bushels of corn, and eight hundred seventy-eight bushels of rye; how many bushels of grain did he raise?

SUBTRACTION.

26. The process of finding the *difference* of two numbers is called **Subtraction.**

27. The **Difference** of two numbers is a number which, added to the less, equals the greater. The difference is also called the **Remainder.**

28. The number to be subtracted is called the **Subtrahend.**

29. The number from which we subtract the other is called the **Minuend.**

30. The **Sign of Subtraction** is $-$, and is read *minus.* Thus, $9 - 6 = 3$ is read "9 minus 6 equals 3," and means that 6 is to be subtracted from 9.

31. To subtract when no term of the subtrahend exceeds the corresponding term of the minuend.

WRITTEN EXERCISES.

1. Subtract 34 from 69.

SOLUTION.—We write the 34 under the 69, draw a line beneath, and begin at the right to subtract. 4 units from 9 units leave 5 units; 3 tens from 6 tens leave 3 tens. The difference is 35.

OPERATION.

$$\begin{array}{r} 69 \\ 34 \\ \hline 35 \end{array}$$

Subtract

2. 425 from 967.
3. 2107 from 8428.
4. 3056 from 4277.
5. 328 from 548.
6. 3072 from 7285.
7. 4208 from 6529.

(8)	(9)	(10)	(11)	(12)	(13)
872	725	857	907	840	876
161	413	654	205	320	345

(14)	(15)	(16)	(17)	(18)	(19)
8769	4876	8275	8799	8591	5857
3257	2142	3251	2542	7230	1234

(20)	(21)	(22)	(23)	(24)
25378	47258	8769	56728	98785
21312	21242	2354	21306	21342

(25)	(26)	(27)	(28)	(29)
373967	873972	72587	95837	89976
212961	132421	51234	51321	32742

ORAL EXERCISES.

1. If Edith had 12 apples and gave 8 of them away, how many apples remained?

SOLUTION.—If Edith had 12 apples and gave 8 of them away, there remained 12 apples minus 8 apples, or 4 apples.

2. Sophia had 15 pins in a cushion, and took out 8 to pin her dress; how many pins remained?

3. There were 20 words given out in the spelling class, and 8 were misspelled; how many were spelled correctly?

4. There were 35 leaves in Ellen's book, and a dog tore out 10 leaves; how many leaves remained in the book?

5. If you cut off 12 feet of a kite-string 25 feet long, how many feet of the string will remain?

6. A boy had 20 marbles, and in playing lost all of them but 5; how many did he lose?

7. There were 25 robins in a cherry tree: a boy shot 8, and the rest flew away; how many flew away?

8. Peter picked 21 peaches for his mother, and his brother took 8 of them; how many remained for his mother?

9. Begin at 20 and count backward by 2's to nothing. Begin at 21 and count backward by 2's to 1.

10. Begin at 21 and count backward by 3's to nothing. Begin at 22 and count backward by 3's to 1. Begin at 23 and count backward by 3's to 2.

11. What is the value of $15 - 7$? Of $20 - 8$? Of $20 - 12$? Of $23 - 11$? Of $24 - 12$? Of $26 - 13$? Of $6 + 8 - 7$? Of $8 + 9 - 10$?

12. What is the value of $8 + 7 - 6$? Of $4 + 9 - 5$? Of $9 + 4 - 6$? Of $8 + 7 - 12$? Of $10 + 11 - 12$?

13. Repeat the table of 5 times; 6 times; 7 times; 8 times; 9 times.

14. Derive the quotients from 5 times; 6 times; 7 times, etc.

WRITTEN EXERCISES.

1. My father had 78 acres of land, and sold 36 acres; how many acres of land remained?

SOLUTION.—If my father had 78 acres of land, and sold 36 acres, there remained the difference between 78 acres and 36 acres, which is 42 acres.

OPERATION.

$$\begin{array}{r} 78 \\ 36 \\ \hline 42 \end{array}$$

2. If there are 86 boys and 54 girls in a school, how many more boys are there in the school than girls?

3. From five thousand seven hundred eighty-five subtract three thousand four hundred forty-three.

4. What is the difference between eight thousand eighty-eight and five thousand seventy-four?

5. A man carted 3047 bricks from a pile containing 8797; how many bricks remained?

6. Mary felt sorry because she couldn't tell how many she must add to 1235 to make 5748; can you tell?

7. What is the difference between twelve thousand seven hundred forty-six and five thousand twenty-five?

8. Sarah had saved four hundred seventy-five cents, and spent two hundred fifty-two cents; how much had she remaining?

9. A farmer raised one thousand eight hundred fifty-eight bushels of corn, and sold five hundred thirty-five bushels; how many bushels remained?

32. To subtract when a term of the subtrahend exceeds the corresponding term of the minuend.

1. Subtract 37 from 64.

SOLUTION.—We write the 37 under 64, draw a line beneath, and begin at the right to subtract. We cannot subtract 7 from 4; so we take 1 ten from the 6 tens, leaving 5 tens, and add the 1 ten to the 4 units, making 14 units; 7 units from 14 units leave 7 units; 3 tens from 5 tens leave 2 tens; hence the remainder is 27.

OPERATION.

64
37
27

NOTE.—In practice name only *results*. The taking 1 from the next term of the minuend is known as *borrowing 10*.

WRITTEN EXERCISES.

(2)	(3)	(4)	(5)	(6)	(7)
63	42	54	51	73	65
25	27	26	24	36	28
(8)	**(9)**	**(10)**	**(11)**	**(12)**	**(13)**
83	86	77	82	58	75
27	38	49	28	29	27
(14)	**(15)**	**(16)**	**(17)**	**(18)**	**(19)**
87	85	76	90	52	63
39	18	67	28	25	36
(20)	**(21)**	**(22)**	**(23)**	**(24)**	**(25)**
342	763	854	981	350	736
218	428	136	245	246	428
(26)	**(27)**	**(28)**	**(29)**	**(30)**	**(31)**
463	734	528	742	585	482
146	382	293	391	129	127
(32)	**(33)**	**(34)**	**(35)**	**(36)**	**(37)**
425	624	735	824	721	456
281	251	143	136	258	183

(38)	(39)	(40)	(41)	(42)	(43)
567	724	812	713	812	732
382	258	365	298	481	458
(44)	(45)	(46)	(47)	(48)	(49)
721	405	713	854	302	620
253	283	265	689	135	154
(50)	(51)	(52)	(53)	(54)	(55)
4236	6874	6528	8257	4167	6458
1218	1236	3184	1821	3725	1823
(56)	(57)	(58)	(59)	(60)	(61)
41365	71624	82571	92051	85601	40701
28134	29131	47138	60423	76543	20630
(62)	(63)	(64)	(65)	(66)	(67)
45060	62500	47000	28000	50000	100000
12345	13287	12345	12379	14625	1

ORAL EXERCISES.

1. Eva culled 20 roses, and gave Floy 12 of them; how many did Eva retain?

2. If there are 20 crows on a tree, and 12 fly away, how many crows are left?

3. A hunter saw 24 rabbits, and shot 9 of them; how many escaped?

4. A boy had 10 cents, and found 12 cents, and then spent 11 cents; how many cents had he then?

5. A boy had 12 apples, picked 6 more, and gave his brother 10; how many apples had he then?

6. Harry bought 20 peaches, gave his sister 8 of them, and ate 6; how many peaches had he then?

7. Begin at 24 and count backward by 4's to naught; begin at 25 and count backward by 4's to 1; begin at 26 and count backward by 4's to 2.

8. Begin at 30 and count backward by 5's to naught; from 31 to 1; from 32 to 2; from 33 to 3; from 34 to 4.

9. What is the value of $8 + 4 - 5$? Of $9 + 4 - 6$? Of $8 + 6 - 9$? Of $7 + 12 - 6$? Of $9 + 7 - 8$?

10. What is the value of $9 - 5 + 4$? Of $8 - 6 + 7$? Of $9 - 5 + 6$? Of $12 - 9 + 10$? Of $20 - 10 + 12$?

WRITTEN EXERCISES.

1. A farmer raised 284 sheep, and sold 156 of them; how many sheep remained unsold?

SOLUTION.—If a farmer raised 284 sheep, and sold 156 of them, there remained the difference between 284 sheep and 156 sheep, which we find, by subtracting, is 128 sheep.

OPERATION.

$$\begin{array}{r} 284 \\ 156 \\ \hline 128 \end{array}$$

2. A lady raised 265 little chickens, and some minks killed 57 of them; how many chickens remained?

3. Frank's kite was 542 feet in the air, and fell 185 feet; how high was it then?

4. One morning in going to school I took 856 steps; how many more should I have to take to make 1000?

5. Emma's brother teased her because she couldn't tell how many to add to 432 to make 500; can you tell?

6. Mary went a-nutting and gathered 180 chestnuts, and gave her little brother 75 of them; how many had she then?

7. Father's new house cost 5860 dollars, and he sold it for 375 dollars less than it cost; how much did he receive for it?

8. A little girl had 54 pins and gave her cousin 27 of them; how many did she have remaining?

9. Fifty little robins were sitting on a tree, and 32 of them flew away; how many were then left?

10. Emma's doll and its little cradle cost 350 cents, and the doll cost 125 cents; how much did the cradle cost?

11. There were 78 roses on two rose-bushes; how many roses were there on the second bush, if there were 39 roses on the first bush?

12. A little girl read 324 words in two days; if she read 146 words one day, how many did she read the other day?

13. Edward and Mary together took 1546 steps in going to school; how many steps did Mary take, if Edward took 738 steps?

14. Minnie had 482 cents in her money-bank, and took out 145 cents to give to a little beggar-girl; how many cents remained?

15. Andrew's kite rose 685 feet, and this was 67 feet higher than Peter's kite went; how high did Peter's kite fly?

16. Charlie wrote 432 words in two weeks; he wrote 246 words the first week; how many words did he write the second week?

17. A boy went to a store and bought a knife for 75 cents, and gave the storekeeper a dollar bill (100 cents) to pay for it; how much change did he receive?

18. Sarah bought a work-box for 275 cents, and gave the storekeeper a 5-dollar bill (500 cents) to pay for it; how much change should she receive?

19. Mr. Nelson's horse and carriage cost four hundred dollars; what did the horse cost, if the carriage cost two hundred and twenty-five dollars?

20. Two little boys picked eighty-four quarts of blackberries one summer; if one picked forty-seven quarts, how many quarts did the other pick?

21. Mr. Barton raised two thousand bushels of wheat and rye; how much rye did he raise, if he raised five hundred and sixty-five bushels of wheat?

WRITTEN EXERCISES

(In Addition and Subtraction).

1. If I have 95 cents in my money-bank, and my uncle puts in 28 cents, how much will be in it then?

2. If Willie writes 185 words this week and 156 words next week, how many words will he write in all?

3. My father had 254 little chickens, and some minks killed 85 of them; how many remained?

4. If I have 216 dollars and give my sister 145 dollars, how much will I have remaining?

5. Mabel's father had 385 acres of land, and sold 178 acres; how many acres did he then have?

6. A lady had 426 dollars, and lent her brother 243 dollars; how much did she then have?

7. I have 365 cents in my money-bank; how many must I put in that there may be 400 cents in it?

8. Sallie had 172 cents, and her brother gave her enough to make her money 234 cents; how much did her brother give her?

9. Carrie's brother teased her because she couldn't tell how much she must add to 245 to make 400; can you tell?

10. Matilda had 120 cents; her mother gave her 236 cents, and then she lent her brother 248 cents; how many cents did she then have?

11. Fannie picked 236 chestnuts, her little brother gave her 78 chestnuts, and she gave 95 to her schoolmates; how many chestnuts remained?

12. One morning in going to school I took 758 steps; how many more steps would I have taken if I had taken 1000 steps in all?

13. Willie's kite was up in the air 536 feet; it then fell 185 feet, and then rose 260 feet; how high was it then?

BUSINESS PROBLEMS.

1. I went to a store and bought a book for 87 cents and a slate for 35 cents; what did I pay for both of them?

2. My mother took me to a store and bought me a top for 15 cents, a cap for 75 cents, and a knife for 45 cents; what did they all cost?

3. William's slate cost 26 cents, his arithmetic 55 cents, his reading-book 48 cents, and his spelling-book 37 cents; what did they all cost?

4. I went to a store and bought a knife for 56 cents, and gave the storekeeper a dollar bill (100 cents) to pay for it; how much change did he give me back?

5. Mary bought a flower-vase for 375 cents, and handed the storekeeper a five-dollar bill (500 cents) to pay for it; how much change should she have received?

6. Mr. Barnes paid 75 dollars for his watch, and sold it so that he gained 12 dollars; what did he receive for it?

7. Martha's new shawl cost 875 cents; if she should sell it so as to gain 125 cents, what would she receive for it?

8. Mr. Taylor's new house cost him 8575 dollars, and he sold it for 735 dollars more than it cost him; what did he receive for it?

9. My father bought a cow for 38 dollars, and sold her for 52 dollars; how much did he gain on the cow?

10. Robert Stewart had a coat which cost him 45 dollars; he sold it to Edward Taylor for 37 dollars; how much did he lose?

11. Harry Hartman sold his watch for 67 dollars, and lost by the sale 15 dollars; what did the watch cost him?

12. Mary's papa gave her a 5-dollar bill to go a-shopping; she bought a fan for 75 cents, some silk for 165 cents, and a pair of gloves for 125 cents; how much change did she bring home?

PROBLEMS ON EVENTS.

WRITTEN EXERCISES.

1. Newspapers were first published in 1630; how long have they been published?

2. Quills were first used for writing about the year 636; how many years ago is that?

3. Cotton was first planted in the United States about the year 1769; how many years ago is that?

4. Glass windows, it is said, were first used in England in 1180; how many years ago is that?

5. Chimneys were first introduced about 1347; how long is it since that time?

6. The potato was taken to England by Sir Walter Raleigh in 1586; how many years has it been in use?

7. Forks were first used about 1458; how long was that before the discovery of America by Columbus?

8. Canary birds were brought to England from the Canary Islands in 1555; how many years since that time?

9. Velocipedes were first invented in 1779; how long is it from that time to 1896?

10. Dr. Jenner introduced vaccination in 1799; how many years has it been in use?

11. Steel pens were introduced in 1820; how many years have they been in use?

12. Starch was introduced into England in 1553; how many years is it since then?

13. Petroleum was discovered in 1859; how many years has it been known?

14. Gas was first used as an experiment in 1792; how long was that before the discovery of petroleum?

15. Calico was first brought to England from India in 1631; how many years ago is that?

16. Some authorities think the Great Pyramid was built 3700 B. C.; how long ago is that?

MULTIPLICATION.

33. The process of finding the *product* of two numbers is called **Multiplication.**

34. The **Product** is the result obtained by taking one number as many times as there are units in another.

35. The number to be multiplied is called the **Multiplicand.** The number by which we multiply is called the **Multiplier.**

36. The **Sign of Multiplication** is $\times$, and is read *times* or *multiplied by.* Thus $4 \times 8 = 32$ is read "4 times $8 = 32$," or "4 multiplied by $8 = 32$."

37. To multiply when the multiplier does not exceed twelve.

1. Multiply 237 by 4.

SOLUTION.—We write the multiplier under the units figure of the multiplicand and begin at the right to multiply.

4 times 7 units are 28 units, which equal 8 units and 2 tens; we write the 8 units in units place, and reserve the 2 tens to add to the next product.

OPERATION.

237
4
948

4 times 3 tens are 12 tens, plus the 2 tens are 14 tens, which equal 4 tens and 1 hundred; we write the 4 tens in tens place and reserve the 2 hundred to add to the next product.

4 times 2 hundreds are 8 hundreds, plus the 1 hundred are 9 hundreds, which we write in hundreds place. The product is 948.

NOTE.—In practice say, 4 times 7 are 28; we write the 8 under the 7 and add the 2 to the next product: 4 times 3 are 12, and 2 are 14; we write the 4 under the 3 and add the 1 to the next product; 4 times 2 are 8, and 1 are 9; we write the 9; the product is 948.

WRITTEN EXERCISES.

(2)	(3)	(4)	(5)	(6)	(7)
54	76	43	56	86	72
3	2	4	5	3	7

(8)	(9)	(10)	(11)	(12)	(13)
85	76	94	38	60	48
2	3	5	7	9	10

(14)	(15)	(16)	(17)	(18)	(19)
50	72	86	80	74	90
11	9	8	12	12	12

(20)	(21)	(22)	(23)	(24)	(25)
234	436	573	682	904	805
3	4	5	6	7	8

(26)	(27)	(28)	(29)	(30)	(31)
728	835	908	736	637	798
6	7	9	8	7	5

(32)	(33)	(34)	(35)	(36)	(37)
970	708	426	654	208	456
8	9	10	11	12	12

Multiply

38. 2175 by 5.	**44.** 89641 by 6.	**50.** 87432 by 9.
39. 3462 by 4.	**45.** 94370 by 4.	**51.** 67546 by 6.
40. 4781 by 6.	**46.** 70842 by 8.	**52.** 89237 by 8.
41. 5847 by 3.	**47.** 68733 by 7.	**53.** 92436 by 10.
42. 6943 by 7.	**48.** 39476 by 9.	**54.** 70406 by 11.
43. 7748 by 8.	**49.** 59473 by 10.	**55.** 83709 by 12.

ORAL EXERCISES.

1. What will 7 yards of cloth cost at 3 dollars a yard?

SOLUTION.—If 1 yard of cloth cost 3 dollars, 7 yards will cost 7 times 3 dollars, or 21 dollars. Therefore, etc.

2. At 5 dollars a day, how much will a man earn in 6 days?

3. What will 5 pairs of boots cost, at 8 dollars a pair?

4. If there are 10 trees in one row, how many trees are there in 8 rows?

5. If a family uses a gallon of milk a day, how many quarts will it use in 8 days?

6. When milk is worth 7 cents a quart, what are 2 gallons of milk worth?

7. How many marbles will Mr. Jones have to buy to give 12 marbles to each of his 5 boys?

8. A fruit-dealer has 6 dozen oranges; how many oranges will he have left after selling 4 dozen?

9. If a boy works 11 examples each day, how many examples will he work in 5 days?

10. There are 12 inches in one foot; how many inches are there in 10 feet?

11. There are 7 days in one week; how many days are there in 12 weeks?

12. If John spends 11 cents a week, how much will he spend in 10 weeks?

13. A newsboy clears 12 cents a day; how much will he clear in 11 days?

14. If a ship sails 12 miles an hour, how far will it sail in 12 hours?

WRITTEN EXERCISES.

1. If 1 yard of muslin costs 25 cents, how much will 7 yards cost?

SOLUTION.—If 1 yard of muslin costs 25 cents, 7 yards will cost 7 times 25 cents, which are 175 cents.

OPERATION.

25 cents.
7
175 cents.

2. If a boy writes 26 words in a day, how many words will he write in 5 days?

3. If a train of cars runs 35 miles an hour, how far will it run in 6 hours?

4. How many marbles will 8 boys have, if each boy has 15 marbles?

5. If Mary can count 75 a minute, how many can she count in 10 minutes?

6. What must a lady pay for 5 dozen eggs at 35 cents a dozen?

7. Alice has 8 rose-bushes in her garden, and on each bush there are 56 roses; how many roses in all?

8. A lady bought 9 pounds of tea at 87 cents a pound; what did it cost her?

9. If there are 36 printed lines on the page of a book, how many printed lines are there in 11 such pages?

10. A storekeeper sells marbles at 15 for a cent; how many can you buy for 7 cents?

11. If Edith attends school 5 hours a day, how many minutes is that, if there are 60 minutes in an hour?

12. There are 6 classes in a school, and 48 pupils in each class; how many pupils in the school?

13. How much must a man pay for 8 loads of hay worth 24 dollars a load?

14. There are 5280 feet in a mile; how many feet are there in 12 miles?

15. Multiply five hundred forty-six by eight.

16. Multiply six hundred seventy-five by nine.

17. Multiply four thousand six hundred eighty-six by six.

18. Multiply seventy-five thousand and twenty-eight by nine.

19. How much will a party of 8 persons pay to ride from Philadelphia to New York, if the fare of each is $2.50?

20. If a girl goes to bed at 9 o'clock at night, and rises at 7 o'clock in the morning, how many hours does she spend in bed in a month of 31 days?

38. To multiply when the multiplier is greater than twelve.

1. Multiply 64 by 23.

SOLUTION.—We write the multiplier under the multiplicand, placing units under units, and tens under tens, and begin at the right to multiply.

3 times 4 units are 12 units, which equal 1 ten and 2 units; we write the units under the 3, and reserve the 1 ten to add to the next product. 3 times 6 tens are 18 tens, and 1 ten added equals 19 tens or 1 hundred and 9 tens, which we write in their proper places.

OPERATION.

64
23
192
128
1472 *Ans.*

Multiplying 64 by 2 in the same manner, we have 128, and since the 2 is 2 *tens*, we have 128 tens, which we write in the proper place; then, adding the two products, we have 1472.

NOTE.—In practice say, 3 times 4 are 12; we write the 2 and carry the 1: 3 times 6 are 18, plus the 1 equals 19, which we write. Then, 2 times 4 are 8, which we write under the 2, and 2 times 6 are 12, which we write beside the 8.

WRITTEN EXERCISES.

(2)	(3)	(4)	(5)	(6)	(7)
35	46	56	65	75	60
23	24	32	34	45	37
(8)	**(9)**	**(10)**	**(11)**	**(12)**	**(13)**
76	43	64	38	85	78
42	38	75	44	76	67
(14)	**(15)**	**(16)**	**(17)**	**(18)**	**(19)**
345	463	547	708	825	756
23	43	35	46	64	63
(20)	**(21)**	**(22)**	**(23)**	**(24)**	**(25)**
725	817	725	809	728	560
45	65	74	86	67	87

(26)	(27)	(28)	(29)	(30)	(31)
2356	4216	2057	3508	7069	4185
35	43	54	63	37	64
(32)	(33)	(34)	(35)	(36)	(37)
345	372	2184	4725	2057	3608
123	345	416	326	354	436
(38)	(39)	(40)	(41)	(42)	(43)
2143	8192	4167	8246	8927	7346
227	426	475	642	352	643
(44)	(45)	(46)	(47)	(48)	(49)
4752	7385	8492	2937	6473	4928
185	218	537	439	567	816
(50)	(51)	(52)	(53)	(54)	(55)
4137	3642	6724	4183	3645	4526
2185	2531	3625	2426	2841	2182

ORAL EXERCISES.

1. How many are 10 times 2? 20 times 2? 30 times 3? 40 times 4? 50 times 5?

2. What two numbers multiplied together make 12? 18? 20? 24? 36? 40? 42? 48? 50? 60? 64?

3. How many words will 12 boys spell, if each boy spells 11 words?

4. In 1 dime there are 10 cents; how many cents are there in 11 dimes?

5. If a man builds 9 rods of wall in a day, how many rods will he build in 12 days?

6. If 12 units make a dozen, how many units are there in 12 dozen?

7. How much more is 12 times 3 than 11 times 2 increased by 3?

8. How many are 2 times 6, plus 6? 3 times 4, plus 4? 4 times 5, plus 5? 6 times 7, plus 7?

9. How many are 2 times 3, plus 4? 3 times 4, plus 5? 4 times 5, plus 6? 5 times 6, plus 7?

10. How many are 3 times 5, plus 6? 6 times 7, plus 8? 7 times 8, plus 9? 9 times 10, plus 11?

11. How many are 4 times 6, minus 8? 6 times 7, minus 5? 8 times 9, minus 10? 7 times 11, minus 9?

12. How many are 6 and 6 times 7? 7 and 7 times 9? 8 and 8 times 10? 9 and 9 times 8?

WRITTEN EXERCISES.

1. If a boat sails 126 miles a day, how far will it sail in 25 days?

SOLUTION.—If a boat sails 126 miles in one day, in 25 days it will sail 25 times 126 miles. Multiplying 126 by 25, we have 3150. Hence the boat will sail 3150 miles in 25 days.

OPERATION.

126 mi.
25
630
252
3150 mi.

2. If a locomotive runs 75 miles a day, how far will it run in 14 days?

3. A man bought a farm containing 24 acres for $250 an acre; how much did it cost?

4. How many trees in a peach-orchard, if there are 22 rows of trees and 46 trees in each row?

5. If a farmer raises 85 bushels of wheat on 1 acre, how many bushels can he raise on 45 acres?

6. A man buys a box of oranges containing 24 dozen; how many oranges are in the box?

7. If in one book there are 364 pages, how many pages in 18 such books?

8. A man bought 326 horses and 36 times as many sheep; how many sheep did he buy?

9. A pencil-manufacturer sold in a month 753 gross

of pencils; since there are 144 pencils in a gross, how many pencils did he sell?

10. A railroad paid its switchmen at an average rate of $79 per month; what was the amount of the wages of 315 switchmen?

11. If your pulse beats 75 times in a minute, how many times will it beat in 4 hours of 60 minutes each?

12. A merchant sold 25 pieces of calico, each containing 35 yards, at 6 cents a yard; what did he receive for his goods?

13. How many minutes in a year of 365 days if there are 1440 minutes in 1 day?

14. A cattle-dealer bought 125 horses, at the rate of 150 dollars apiece; how much did they cost him?

15. There are 5760 grains in one pound Troy; how many grains in 245 pounds Troy?

39. To multiply when one or both terms contain ciphers.

1. Multiply 537 by 204.

SUGGESTION.—We first multiply by 4, obtaining 2148; we then pass over the naught and multiply by 2, placing the right-hand figure of the product directly under the 2. Adding, we have 109548.

OPERATION.

537
204
2148
1074
109548

WRITTEN EXERCISES.

Multiply

2. 367 by 305.
3. 528 by 403.
4. 709 by 406.
5. 596 by 307.
6. 854 by 508.
7. 4325 by 3004.
8. 2306 by 2005.
9. 5762 by 4006.
10. 3857 by 6054.
11. 7205 by 3506.
12. 8096 by 4008.
13. 7084 by 5007.

14. Multiply 4600 by 340.

SUGGESTION.—We first multiply 46 by 34, and obtain the product 1564; we then annex the three ciphers to this product, and we have 1564000. The teacher will explain the reason for the operation.

OPERATION.

4600
340
184
138
1564000

Multiply—

15. 4500 by 240.
16. 3600 by 650.
17. 7200 by 380.
18. 8500 by 4200.
19. 6510 by 7300.
20. 4700 by 3500.
21. 3060 by 2050.
22. 7000 by 3050.
23. 7080 by 8000.
24. 30600 by 50700.

WRITTEN EXERCISES.

1. How much will 68 barrels of flour cost, at 6 dollars a barrel?

SOLUTION.—If 1 barrel of flour cost 6 dollars, 68 barrels of flour will cost 68 times 6 dollars, which are 408 dollars.

OPERATION.

68
6
$408

NOTE.—In practice we multiply the 68 by 6, since it is more convenient to use the smaller number as the multiplier.

2. How much will 75 bushels of potatoes cost, at 48 cents a bushel?

3. How much will 85 bushels of corn cost, at 52 cents a bushel?

4. How much will 65 yards of cambric cost, at 37 cents a yard?

5. A drover bought 296 sheep, at 9 dollars each; what did they cost?

6. There are 231 cubic inches in a wine gallon; how many cubic inches in 56 wine gallons?

7. There are 5280 feet in a mile; how many feet are there in 156 miles?

8. There are 4840 square yards in one acre; how many square yards in 365 acres?

DIVISION.

40. The process of finding the *quotient* of two numbers is called **Division.**

41. The **Quotient** of two numbers is the number which shows how many times one number contains another.

42. The number to be divided is called the **Dividend.**

43. The number by which we divide is called the **Divisor.**

44. The part of the dividend which sometimes remains is called the **Remainder.**

45. The **Sign of Division** is $\div$, and is read *divided by.* It shows that the number on the left is to be divided by the number on the right. Thus, $20 \div 5 = 4$ is read "20 divided by 5 equals 4."

46. Division is also indicated by writing the divisor under the dividend, with a line between them. Thus, $\frac{20}{5}$ means 20 divided by 5.

47. Division is also indicated by writing the divisor at the left of the dividend with a curved line between them. Thus, 5)20 means 20 divided by 5.

48. To divide by the method of Short Division.

1. Divide 348 by 6.

SOLUTION.—6 is contained in 34 tens 5 tens times and 4 tens remaining; we write the 5 in tens place; 4 tens and 8 units are 48 units; 6 is contained in 48 units 8 units times; we write the 8 in units place. The quotient is 58.

OPERATION.

$$6)\underline{348}$$
$$58$$

1. In practice the pupil may say, 6 is contained in 34, 5 times and 4 remaining; 6 is contained in 48, 8 times. Hence the quotient is 58.

2. A final remainder may be set off by itself at the right of the quotient.

WRITTEN EXERCISES.

(2)	(3)	(4)	(5)	(6)	(7)
2)328	2)573	2)756	3)450	3)524	3)678
(8)	**(9)**	**(10)**	**(11)**	**(12)**	**(13)**
3)535	3)795	3)468	4)852	4)704	4)535
(14)	**(15)**	**(16)**	**(17)**	**(18)**	**(19)**
5)651	5)848	5)780	5)875	5)735	5)944
(20)	**(21)**	**(22)**	**(23)**	**(24)**	**(25)**
6)858	6)785	6)659	6)853	6)786	6)6854
(26)	**(27)**	**(28)**	**(29)**	**(30)**	**(31)**
7)854	7)952	7)998	7)784	7)847	7)9807
(32)	**(33)**	**(34)**	**(35)**	**(36)**	**(37)**
8)433	8)447	8)736	8)4568	8)5672	8)6465
(38)	**(39)**	**(40)**	**(41)**	**(42)**	**(43)**
9)702	9)585	9)850	9)3672	9)4624	9)7264

ORAL EXERCISES.

1. How many trees in an orchard, if there are 8 rows and 10 trees in each row?

2. A man bought 8 parrots at 4 dollars each; how much did he pay for them?

3. At the rate of 5 dollars apiece, how many sheep can be bought for 50 dollars?

4. There are 120 trees in an orchard, and 10 trees in each row; what is the number of rows?

5. A fly has 6 legs and 2 wings; how many legs and wings have 12 flies?

6. If 72 books are arranged in piles of 8 books each, how many piles will there be?

7. How many nails will be required for 64 horseshoes, if it takes 8 nails for each horseshoe?

8. How many times can a five-gallon can be filled from a barrel containing 32 gallons of kerosene?

9. A farmer's wife, putting up eggs for market, found she had 87; how many dozen had she, and how many over?

10. A postman finds that in delivering mail he walks 18 miles a day; if he makes 3 trips a day, how far does he walk each trip?

11. George added columns containing 80 figures, having 10 figures in a column; how many columns did he add?

12. James paid $21 for tickets for a party to New York by a Sound steamboat; if there were 7 in the party, what was the price of a ticket?

13. Read the proper numbers in place of (?).

$32 + 6 = ?$	$36 + 9 = ?$	$48 + 8 = ?$
$24 - 6 = ?$	$36 - 9 = ?$	$48 - 8 = ?$
$14 \times 6 = ?$	$13 \times 9 = ?$	$14 \times 8 = ?$
$24 \div 6 = ?$	$36 \div 9 = ?$	$48 \div 8 = ?$

WRITTEN EXERCISES.

1. How long will it take a steamboat to sail 272 miles, at the rate of 8 miles an hour?

SOLUTION.—It will take as many hours as 8 is contained times in 272, which are 34.

OPERATION.

$$8\overline{)272}$$
$$34$$

2. How many pigs can a farmer buy for 238 dollars, at the rate of 7 dollars apiece?

3. How long would it take a boy to walk 236 miles, at the rate of 4 miles an hour?

4. How many yards of velvet can a tailor buy for 435 dollars, at the rate of 5 dollars a yard?

5. How many barrels of coal oil will 516 dollars buy, at the rate of 6 dollars a barrel?

6. If a boy writes 5 words a minute, how many minutes would it take him to write 345 words?

7. If a man pays 7 dollars a week for board, how many weeks could he board for 343 dollars?

8. If a boat sails 9 miles an hour, how long would it take her to sail 729 miles?

9. How many jackets, at 5 dollars each, could be bought for 825 dollars?

10. The amount of salaries paid 6 teachers in a certain school was 3600 dollars; what was the average salary?

11. The amount taken up at a church collection was 4296 cents, and the average amount for each person present was 8 cents; how many were present?

12. How many tons of coal can be bought for 642 dollars, at 6 dollars a ton?

13. How many fathoms deep is the sea at a depth of 7254 feet, there being 6 feet in a fathom?

14. If a man earns 8 dollars a week, how long will it take him to earn 672 dollars?

15. A farmer sold 580 eggs; how many dozen did he sell, and how many over?

16. How many times can an 8-quart measure be filled from 250 quarts? How much will be left?

17. How many times will a bicycle wheel, 12 feet in circumference, revolve in going one mile, or 5280 feet?

18. There are 7 days in one week; how many weeks are there in 560 days?

19. There are 12 inches in one foot; how many feet are there in 1728 inches?

20. A yacht sailed up a river for 14 hours at the rate of 6 miles an hour, but it came down with the current in 12 hours; what was its rate in coming down?

49. To divide by the method of Long Division when the divisor exceeds twelve.

1. Divide 455 by 13.

SOLUTION.—13 is contained in 45 tens 3 tens times; 3 tens times 13 are 39 tens; 39 tens subtracted from 45 tens leave 6 tens; bringing down the 5 units, we have 65; 13 is contained in 65, 5 times; 5 times 13 are 65; subtracting, nothing remains. The quotient is 35.

OPERATION.

```
13)455(35
   39
   ---
   65
   65
   ---
```

1. The teacher will call the pupil's attention to the fact that there are four operations: 1st, *divide;* 2d, *multiply;* 3d, *subtract;* 4th, *bring down.*

2. If, when we multiply, the product is greater than the partial dividend, the quotient term is too large, and must be diminished.

3. When a remainder is equal to or greater than the divisor, the quotient term is too small, and must be increased.

4. A final remainder may be set off by itself, or it may be written over the divisor and annexed to the quotient.

5. Some teachers have the pupils form a table of the products of the divisor multiplied by each of the first nine numbers, before beginning to divide; but it is of doubtful propriety.

Divide

2. 374 by 11.
3. 744 by 12.
4. 364 by 13.
5. 490 by 14.
6. 795 by 15.
7. 896 by 16.
8. 792 by 17.
9. 534 by 18.
10. 684 by 19.
11. 760 by 20.
12. 3780 by 21.
13. 4862 by 22.
14. 2622 by 23.
15. 5760 by 24.
16. 4925 by 25.
17. 5460 by 26.
18. 9362 by 31.
19. 7317 by 32.
20. 5941 by 33.
21. 6330 by 41.
22. 8670 by 42.
23. 5184 by 43.
24. 4368 by 51.
25. 8568 by 52.
26. 7534 by 53.
27. 8746 by 54.

NOTE.—The teacher will explain how to proceed when there are 0's in the quotient.

ORAL EXERCISES.

1. How many settees will seat 72 boys, if there are 8 boys on each settee?

2. A boy has 80 marbles; how many groups of 10 marbles can he make out of them?

3. How many are 12 plus 6, divided by 6? 25 plus 5, divided by 5? 40 plus 8, divided by 8?

4. How many are 16 and 4, divided by 4? 18 and 6, divided by 6? 35 and 5, divided by 5?

5. How many are 40 minus 4, divided by 4? 50 minus 5, divided by 5? 60 minus 6, divided by 6?

6. How many are 6 times 4, divided by 4? 5 times 8, divided by 8? 6 times 8, divided by 4? 9 times 8, divided by 6?

WRITTEN EXERCISES.

1. At 14 dollars a ton, how many tons of hay can I buy for 560 dollars?

SOLUTION.—I can buy as many tons as \$14 is contained times in \$560, which are 40 tons.

OPERATION.

14)560(40

2. How many cows can a farmer buy for 1680 dollars, at 24 dollars apiece?

3. In 1 pound there are 16 ounces; how many pounds are there in 208 ounces?

4. In 1 bushel there are 32 quarts; how many bushels are there in 480 quarts?

5. If a train of cars runs 35 miles an hour, how long will it take to run 770 miles?

6. How many days are there in 7200 hours, there being 24 hours in a day?

7. If the product of two numbers is 3825, and one of the numbers is 45, what is the other number?

8. If you weigh 1216 ounces, how many pounds do you weigh, 16 ounces being equal to 1 pound?

9. If you weigh 76 pounds, and your sister weighs 112 ounces less, how many pounds does she weigh?

10. There are 800 trees in an orange-grove, arranged in 32 rows; how many trees are there in each row?

11. Willis's father bought 36 packs of fire-crackers, costing 144 cents; what was the price of a pack?

12. If a boy rides on his bicycle 300 miles in 25 days, how far does he ride on an average in a day?

13. A storekeeper sold 48 spools of thread for $1.92; what was the price per spool?

14. A clothier sold 42 overcoats for $882; what was the average price for each overcoat?

15. A family used 32 pounds of butter a month, at a cost of $10.24; what was the price per pound?

16. The diameter of the earth is nearly 8000 miles; how long would it take a boy to walk this distance if he walked 32 miles a day?

50. Exercises in which the divisor is larger than in the previous examples.

WRITTEN EXERCISES.

Divide

1. 2862 by 54.
2. 3630 by 55.
3. 1960 by 56.
4. 7872 by 61.
5. 4464 by 62.
6. 5544 by 63.
7. 2944 by 64.
8. 8051 by 71.
9. 3240 by 72.
10. 9709 by 73.
11. 5032 by 74.
12. 9716 by 81.
13. 9430 by 82.
14. 7055 by 83.
15. 26292 by 84.
16. 37674 by 91.
17. 25692 by 92.
18. 60543 by 93.
19. 13536 by 94.
20. 47785 by 95.
21. 26814 by 123.
22. 49500 by 132.
23. 71288 by 142.
24. 92736 by 166.

25. What number, multiplied by 143, will give a product of 13728?

26. Mr. Thomas wishes to invest $1458 in land; how much can he buy at $54 an acre?

27. I bought 65 shares of bank stock for $3250; what was the cost of a share?

28. A drover sold a herd of 56 cows for $1960; what was the average price per cow?

29. How long would it take a yacht to cross the Atlantic, about 3000 miles, at the rate of 75 miles a day?

30. The distance around the world is nearly 25000 miles; how long would it take to sail around it if the ship sailed 125 miles a day?

51. To divide when there are ciphers on the right of the divisor.

WRITTEN EXERCISES.

1. Divide 2786 by 500.

SOLUTION.—We cut off the two ciphers of 500, and the two terms 86, and divide 27 by 5; we have a quotient of 5, and 2 remaining; we write the 2 and annex the 86 to it, and have 286 for the entire remainder.

OPERATION.

$5|00)27|86$

$5\text{-}286$

or $5\frac{286}{500}$.

NOTE.—When the divisor, with ciphers cut off, is greater than 12, we divide by the method of long division.

Divide

2. 174 by 50.
3. 297 by 60.
4. 476 by 70.
5. 960 by 80.
6. 1225 by 400.
7. 7642 by 600.
8. 8890 by 700.
9. 12476 by 800.
10. 4344 by 150.
11. 6274 by 160.
12. 3872 by 220.
13. 54360 by 240.
14. 18754 by 2500.
15. 54437 by 3200.
16. 114720 by 4500.
17. 202040 by 12200.

WRITTEN EXERCISES.

1. There are 16 ounces in 1 pound; how many pounds in 3600 ounces?

2. In 1 bushel there are 32 quarts; how many bushels in 7872 quarts?

3. A drover gave 1975 dollars for cows worth 25 dollars each; how many did he buy?

4. A horse-dealer received $16,500 for a lot of horses sold at $150 each; how many horses did he sell?

5. How long will it take a vessel to sail 6460 miles, at the rate of 68 miles a day?

6. How many bushels of wheat, at 56 cents a bushel, can be bought for 13272 cents?

7. There are 2583 gallons in 41 hogsheads; how many gallons are there in 1 hogshead?

8. How many carriages can you buy for 8832 dollars, at the rate of 184 dollars each?

9. If sound moves 1092 feet in a second, in how many seconds will it move 34944 feet?

10. The moon is about 240,000 miles from the earth; how long would it require a cannon-ball to reach it, moving at the rate of 48 miles a minute?

PRACTICAL PROBLEMS

(In Fundamental Operations).

1. Find the sum of six hundred five and eighteen hundred ninety-seven.

2. Subtract one thousand nineteen from four thousand seventy-five.

3. A farmer had 397 pigs, and bought 85 pigs, and then sold 182 pigs; how many had he then?

4. Henry had 756 dollars, and his mother gave him enough to make his money 1200 dollars; how much did his mother give him?

5. Mary's father left her 596 acres of Western land: she sold 484 acres, and then bought 396 acres; how many acres had she then?

6. A man deposited \$8000 in the bank; he drew out at one time \$2575, at another \$3467, at another \$1576; how much remained in the bank?

7. My barn cost \$2318, my house cost 3 times as much, and my farm as much as both; what was the cost of the house? the cost of the farm?

8. A clerk receives a salary of \$75 a month; he spends \$18 a month for board, and \$9 for other expenses; how much can he save in a year?

9. Thomas travels 24 miles a day, and Walton travels 52 miles a day; how much farther does Walton travel in 72 days than Thomas?

10. A farmer, having \$3420, bought 35 cows at \$24 a head, and 36 oxen at \$54 a head; how much has he left after paying for them?

11. A man, having \$18000, leaves his wife \$4800, and divides the remainder equally among 6 children; what does each receive?

12. The income of a man who "struck oil" is \$400 per day; how many teachers would this employ at a salary of \$730 a year?

13. If the President of the United States expends 104 dollars daily, how much can he save in a year of 365 days out of his salary of 50000 dollars.

14. If the Vice-President expends 35 dollars daily, how much can he save in a year if he has an income of \$6450 besides his salary of \$8000 a year?

15. If the Secretary of State expends 16 dollars a day, how much can he save in a year, his salary being \$8000 a year and his private income \$28 a week?

PROBLEMS ON AMERICAN HISTORY.

1. America was discovered in 1492, and Jamestown was settled in 1607; required the difference of time.

2. Plymouth was settled in 1620; how long was that after the discovery of America?

3. Philadelphia was founded by William Penn in 1682; how many years old is the city?

4. The battle of Lexington was fought in 1775; how long was that after the settlement of Plymouth?

5. The Declaration of Independence was made in 1776; how long is it since then?

6. The battle of Bunker Hill was fought in 1775; how long was this after the settlement of Jamestown?

7. The surrender of Burgoyne at Saratoga occurred in 1777; how long is it since then?

8. Gen. Cornwallis surrendered at Yorktown in 1781; how many years is it since then?

9. The inauguration of Washington took place in 1789; how long ago was that?

10. The battle of New Orleans took place in 1815; how long was that after the inauguration of Washington?

11. The frigate Constitution captured the British frigate Guerriere in 1812; how long ago was that?

12. Commodore Perry won his great naval victory in 1813; how long was that after the battle of Lexington?

13. General Jackson won the battle of New Orleans in 1815; how long ago was that?

14. The battle of Buena Vista was fought in 1847; how long was that after the battle of New Orleans?

15. Fort Sumter was fired upon in 1861; how long was that after the Declaration of Independence?

16. The Civil War ended in 1865; how long is it from that time till the present?

PROBLEMS IN EUROPEAN HISTORY.

1. Rome was founded 753 years before the Christian era; how many years is it since then?

2. The Roman empire was divided 395 A.D.; how long was that after the founding of Rome?

3. The Romans invaded Britain 55 years before the Christian era; how long ago was that?

4. The Saxons invaded Britain in 449; how long was this after the first invasion of the Romans?

5. King Alfred died in the year 901; how long was that after the invasion of the Romans?

6. William the Conqueror won the battle of Hastings in 1066; how long was that after the death of Alfred?

7. Robert Bruce won the battle of Bannockburn in 1314; how long was that after the battle of Hastings?

8. Queen Elizabeth ascended the English throne in 1558; how long was that after the death of Alfred?

9. The defeat of the Spanish Armada took place in 1588; how long was that after the battle of Hastings?

10. James I. became king of England in 1603; how long was that after the defeat of the Armada?

11. Charles I. was beheaded in 1649; how long was that after the battle of Bannockburn?

12. George III. became king in 1760; how long was that before the Declaration of Independence?

13. Louis XVI. of France was beheaded in 1794; how long was that after the battle of Hastings?

14. Napoleon Bonaparte was born in 1769; how long was that after the birth of Charlemagne in 742?

15. The battle of Waterloo was fought in 1815; how long was that after the birth of Napoleon?

16. Queen Victoria ascended the English throne in 1837; how long has her reign continued?

Queen Victoria died

INTRODUCTION TO FACTORING.

52. Pupils can be led to the ideas of this section as is indicated by the following questions:

1. What numbers multiplied together will *produce* 4? 6? 8? 10? 12? 14? 16? 18? 20? 24?

2. What numbers can be *composed* out of the numbers 2 and 3? 3 and 5? 2, 3, and 5? 3, 4, and 5? 3, 5, and 7?

3. Will the product of any two numbers, each greater than a unit, produce 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, and 37?

4. What may we call a number which is *composed* by multiplying several numbers together? *Ans.* A *Composite Number.*

5. What shall we call numbers that cannot be produced by multiplying several numbers together? *Ans. Prime Numbers.*

6. Which are prime and which composite numbers in the following list: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15?

7. What may we call the numbers whose product *makes* a composite number? *Ans. Makers* of the number.

8. If the word *Factor* means the same as *maker*, what may we call the *makers* of a composite number? *Ans. Factors.*

9. What are the factors of 12? Of 15? Of 18? Of 20? Of 21? Of 24? Of 27? Of 30? Of 32? Of 36?

10. Form a composite number by using 2 twice as a factor; 3 twice as a factor; 2 three times as a factor; 3 four times as a factor.

11. What would it seem natural to call the process of making *composite* numbers? *Ans. Composition.*

12. What would it seem natural to call the process of finding the *factors* of a number? *Ans. Factoring.*

13. Name the prime numbers which are factors of 12; of 18; of 20; of 24; of 27; of 36.

14. What shall we call the factors of numbers when they are prime numbers? *Ans. Prime Factors.*

15. What divisor is common to 4 and 6? To 6 and 9? To 8 and 12? To 12 and 16?

16. What may a divisor common to two or more numbers be called? *Ans.* Their *Common Divisor.*

17. What may the greatest divisor common to two or more numbers be called? *Ans.* Their *Greatest Common Divisor.*

SECTION VI.

COMPOSITION AND FACTORING.

53. A number that is the product of two or more numbers, each greater than a unit, is called a **Composite Number.**

Thus, 6 is a composite number, since it is the product of 3 and 2, each of which is greater than a unit.

54. A number that is not the product of two or more numbers, each greater than a unit, is called a **Prime Number.**

Thus, 2, 3, 5, and 7 are prime numbers, since they are not the products of any numbers, each greater than a unit.

55. The numbers which produce a composite number are called the **Factors** of the number.

Thus, 2 and 3 are the factors of 6, since 3 times 2 are 6; 4 and 3 are the factors of 12, since 4 times 3 are 12.

56. When the factors of a number are prime numbers they are called **Prime Factors.**

Thus, 2, 3, and 5 are the prime factors of 30.

ORAL AND WRITTEN EXERCISES.

1. What numbers multiplied together will produce 8, 10, 12, 14, 15, 18, 20, 24, 33, 72, 84, 108?

2. What are the factors of 10, 14, 15, 18, 21, 24, 25, 27, 28, 32, 33, 42, 55, 75, 96, 144, 216?

3. What prime numbers multiplied together will produce 6, 8, 12, 15, 16, 18, 20, 22, 24, 28, 35, 40, 56, 74?

4. What are the prime factors of 12, 18, 24, 27, 36, 40, 48, 64, 72, 80, 96, 132, 144?

COMPOSITION.

57. The process of composing numbers out of their factors is called **Composition.**

Thus, the producing of 12 out of its factors, 3 and 4, is composition.

Every composite number is equal to the product of its prime factors.

ORAL EXERCISES.

1. What composite number is produced by the two factors, 3 and 5?

2. What composite number is produced by the three factors, 2, 3, and 7?

3. Find the composite number composed of three 2's and two 3's.

4. Find the composite number composed of three 2's, two 3's, and one 5.

5. Make all the composite numbers you can out of 2, 3, and 5.

WRITTEN EXERCISES.

1. Find the composite number whose factors are 2, 3, and 5.

SOLUTION.—To find the composite number whose factors are 2, 3, and 5, find the product of these factors. 5 multiplied by 3 is 15, and 15 multiplied by 2 is 30. Hence the composite number is 30. Hence to form a composite number from its factors *find the product of all the factors.*

OPERATION.

$$\begin{array}{r} 5 \\ \times\ 3 \\ \hline 15 \\ \times\ 2 \\ \hline 30 \end{array}$$

2. Find the composite number composed of the factors 3, 5 7, and 11.

3. Find the composite number composed of the factors 7, 9, 15, and 19.

4. Find the composite numbers composed of two equal factors, when each is 35; each 57; each 109; each 163.

5. Find the composite numbers consisting of three equal factors, each being 7; each 11; each 17; each 25.

6. Find the composite numbers consisting of four equal factors, each being 5; each 13; each 15; each 24.

7. Find the composite number produced by the five smallest prime numbers.

8. Find the composite number produced by the five smallest composite numbers.

9. Make three composite numbers out of the prime factors 2, 3, and 5.

10. Make all the composite numbers you can out of the factors 3, 5, 7, 11.

FACTORING.

58. The process of finding the factors of composite numbers is called **Factoring.**

Thus, the finding of the factors, 3 and 4, of 12, is factoring.

Every composite number is divisible by its prime factors.

Thus, 15, which is the product of its two prime factors, 3 and 5, is divisible by 3 and by 5.

ORAL EXERCISES.

1. Name the factors of 4; of 6; of 8; of 9; of 10; of 12; of 14; of 15.

2. Name the prime factors of 8; of 12; of 18; of 20; of 24; of 27; of 30.

3. Name four numbers exactly divisible by 2. Name four numbers not exactly divisible by 2.

4. Show that each number in which the units figure is 2, 4, 6, 8, or 0, is divisible by 2.

5. Name four numbers exactly divisible by 3. Name four numbers not exactly divisible by 3.

6. What is the sum of the digits in 27? In 45? In 234? Are these numbers divisible by 3?

7. Show that any number in which the sum of the digits is divisible by 3 is itself divisible by 3.

8. Show that any number in which the unit figure is 5 or 0 is divisible by 5.

9. Which of the following numbers are divisible by 2, by 3, and by 5: 26, 42, 45, 60, 72, 75, 123, 132?

59. The following principles, illustrated above, will aid pupils in finding the factors of numbers:

1. *A number is exactly divisible by* 2 *when the unit figure is an even number or* 0.

2. *A number is exactly divisible by* 3 *when the sum of its digits is divisible by* 3.

3. *A number is exactly divisible by* 5 *when its right hand figure is* 5 *or* 0.

WRITTEN EXERCISES.

1. What are the prime factors of 60?

SOLUTION.—Dividing 60 by 2, we have a quotient of 30; dividing 30 by 2, we have a quotient of 15; dividing 15 by 3, we have a quotient of 5; hence 2, 2, 3, and 5 are the factors of 60, and since they are prime numbers, they are the prime factors of 60. Hence the method:

OPERATION.

$$\begin{array}{r} 2)60 \\ 2)30 \\ 3)15 \\ 5 \end{array}$$

I. *Divide the number by any prime number, except* 1, *that will exactly divide it.*

II. *Divide the quotient, if composite, in the same manner, and thus continue until the quotient is a prime number.*

III. *The divisors and the last quotient are the prime factors required.*

Find the prime factors of

2. 48.	**6.** 175.	**10.** 475.	**14.** 1200.
3. 72.	**7.** 270.	**11.** 858.	**15.** 7290.
4. 81.	**8.** 315.	**12.** 1575.	**16.** 29295.
5. 108.	**9.** 336.	**13.** 8316.	**17.** 80850.

GREATEST COMMON DIVISOR.

60. An integer that will divide a number without a remainder is called an **Exact Divisor** of the number.

Thus, 4 is an exact divisor of 12, since it divides 12 without a remainder.

61. A **Common Divisor** of two or more numbers is an exact divisor of each of them.

Thus, 4 is a common divisor of 16 and 24, since it exactly divides each of them.

62. The **Greatest Common Divisor** of two or more numbers is the greatest exact divisor of each of them.

Thus, 18 is the greatest common divisor of 36 and 54, since it is the greatest number that will exactly divide each of them.

ORAL EXERCISES.

1. Name some divisors of 8; of 12; of 18; of 24; of 36; of 40; of 45.

2. What factors are common to 8 and 12? 9 and 12? 20 and 30? 24 and 36?

3. What divisors are common to 12 and 16? 18 and 24? 36 and 48? 50 and 60?

4. What do we call divisors that are common to two or more numbers?

5. Name the common divisors of 12 and 24; of 16 and 20; of 20 and 36.

6. What is the largest divisor common to 8 and 12? to 12 and 14? to 12 and 16? to 24, 36, and 72? to 25, 50, and 125?

63. From these exercises it will be seen that *the greatest common divisor of two or more numbers equals the product of all the common prime factors of those numbers.*

WRITTEN EXERCISES.

1. Find the greatest common divisor of 24, 30, and 42.

SOLUTION.—The factors of 24 are 2, 3, and 4; the factors of 30 are 2, 3, and 5; the factors of 42 are 2, 3, and 7. The *common factors* of 24, 30, and 42 are 2 and 3; hence the product of 2 and 3, or 6, is the greatest common divisor of 24, 30, and 42. Hence the following method:

OPERATION.

$24 = 2 \times 3 \times 4$
$30 = 2 \times 3 \times 5$
$42 = 2 \times 3 \times 7$
$2 \times 3 = 6$

Resolve the numbers into their prime factors, and take the product of all the common prime factors.

Find the greatest common divisor

2. Of 30 and 36.
3. Of 60 and 90.
4. Of 44 and 66.
5. Of 96 and 84.
6. Of 175 and 245.
7. Of 12, 15, and 21.
8. Of 18, 24, and 36.
9. Of 36, 72, and 108.
10. Of 84, 126, and 210.
11. Of 546, 630, and 1638.

12. What is the length of the longest pole with which you can measure 126 ft., 144 ft., and 156 ft.?

13. Three pieces of carpet, of 48, 64, and 80 yards, will exactly cover Mrs. White's parlor if cut into the longest possible equal lengths. How long is the parlor?

14. A man owns three tracts of land on a street whose fronts measure 216 ft., 240 ft., and 336 ft. respectively; what is the front of the largest lots of equal size into which he can divide them?

15. A farmer has 225 bushels of oats and 325 bushels of rye, which he wishes to put in bins of equal size containing the greatest possible number of bushels; what will be the contents of a bin?

16. A has \$240, B \$300, and C \$450, with which they agree to buy cows at such a rate that each man can invest all his money; what is the highest price they can pay per head?

LEAST COMMON MULTIPLE.

64. A **Multiple** of a number is one or more times that number.

Thus, 12 is a multiple of 4, since it is *three* times 4.

65. A **Common Multiple** of two or more numbers is a number which is a multiple of each of them.

Thus, 24 is a common multiple of 4 and 6, since it is a number of times each of them.

66. The **Least Common Multiple** of two or more numbers is the least number which is a multiple of each of them.

Thus, 12 is the least common multiple of 4 and 6, since it is the least number that is a number of times each of them.

ORAL EXERCISES.

1. What number is a multiple of 3? Of 4? Of 5? Of 6? Of 7? Of 8?

2. Name two multiples of 8; two multiples of 10; three multiples of 9; three multiples of 12.

3. What number is a multiple of both 4 and 6? Of 5 and 6? Of 6 and 8?

4. What may we call a multiple which is *common* to two or more numbers?

5. Name a common multiple of 3 and 4; of 6 and 9; of 8 and 12; of 9 and 12.

6. What is the smallest number that is a multiple of 4 and 8? Of 4 and 6? Of 6 and 8? Of 8 and 12?

7. What may we call the *least* multiple *common* to two or more numbers?

8. Name the least common multiple of 4 and 6; of 4 and 8; of 6 and 8; of 8 and 10; of 9 and 12.

67. From these exercises it will be seen that *the least common multiple of two or more numbers must contain all the factors of each number, and no other factors.*

WRITTEN EXERCISES.

1. Find the least common multiple of 6 and 15.

SOLUTION.—The prime factors of 6 are 2 and 3; hence a multiple of 6 must contain the factors 2 and 3. The factors of 15 are 3 and 5; hence a multiple of 15 must contain the additional factor, 5. The least common multiple, therefore, of 6 and 15 is $2 \times 3 \times 5$, or 30. Hence the following method:

OPERATION.

$6 = 2 \times 3$

$15 = 3 \times 5$

L. C. M. $= 2 \times 3 \times 5 = 30$

Resolve the numbers into their prime factors, and take the product of all the different factors, using each factor the greatest number of times it occurs in either number.

Find the least common multiple

2. Of 12 and 15.
3. Of 15 and 18.
4. Of 16 and 18.
5. Of 48 and 72.
6. Of 27 and 135.
7. Of 6, 8, and 10.
8. Of 9, 12, and 15.
9. Of 15, 18, and 24.
10. Of 84, 96, and 108.
11. Of 63, 105, and 189.

12. At a Sunday-school collection in one class each member gave 5 cents; in another, 6 cents; in another, 8 cents; and in the fourth, 10 cents; and each class contributed the same amount: what is the least sum with which this could happen?

13. The piece-goods in a case of silk are to cut without waste into dress patterns of either 12, 15, 20, or 30 yards: what are the shortest lengths into which the piece-goods can be made?

CANCELLATION.

68. A process of shortening computations by rejecting common factors from the dividend and divisor is called **Cancellation.**

PRINCIPLE.—*Cancelling a common factor from both dividend and divisor does not change the quotient.*

For if we divide 18 by 6, the quotient is 3; and also, if we resolve 18 and 6 into their factors, and cancel the common factor, 3, the quotient is then 3.

$$\frac{18}{6} = 3$$

$$\frac{\cancel{3} \times 6}{2 \times \cancel{3}} = 3$$

WRITTEN EXERCISES.

1. Divide 28 by 8.

SOLUTION.—Write the divisor, 8, under the dividend, 24. Resolve 28 into the factors, 4×7, and 8 into 2×4; then cancel the common factor, 4, in the dividend and divisor, and we have 7 divided by 2, or $3\frac{1}{2}$. Hence we have the following method:

OPERATION.

$$\frac{28}{8} = \frac{\cancel{4} \times 7}{2 \times \cancel{4}} = \frac{7}{2} = 3\tfrac{1}{2}$$

I. *Cancel the common factors from the dividend and divisor.*

II. *Then divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor.*

NOTE.—When a factor is cancelled, the unit, 1, takes its place, but need not be written, except in the quotient when there are no other factors.

Divide:

2. 48 by 30.
3. 54 by 45.
4. 72 by 63.
5. 42 by 30.
6. 90 by 50.
7. 144 by 120.
8. $4 \times 5 \times 6$ by 60.
9. $5 \times 7 \times 12$ by 5×6.
10. $4 \times 6 \times 8$ by $3 \times 5 \times 7$.
11. $12 \times 14 \times 18$ by $6 \times 7 \times 9$.
12. Divide $7 \times 9 \times 10$ by $3 \times 5 \times 7$.
13. Divide $8 \times 10 \times 12$ by $4 \times 5 \times 16$.

14. Divide $27 \times 12 \times 14$ by $9 \times 4 \times 7$.

15. Divide $72 \times 45 \times 140$ by $18 \times 24 \times 35$.

16. Divide $35 \times 36 \times 48$ by $30 \times 27 \times 40$.

17. How many cows, at \$20 each, can be obtained for 12 horses, at \$90 each?

18. How many pigs, at 5 dollars each, can be obtained for 20 barrels of corn, at 3 dollars a barrel?

19. A miller gave 25 barrels of flour, at \$6 a barrel, for hay, worth \$15 a ton; how many tons did he receive?

20. How many bushels of oats, worth 55 cents a bushel, can be exchanged for 44 bushels of rye, at 75 cents a bushel?

21. A lady bought 15 yards of ribbon, at 48 cents a yard, and exchanged it for other ribbon, at 45 cents a yard; how many yards did she get?

22. A farmer exchanged 20 bushels of wheat, at 84 cents a bushel, for cloth, worth \$1.05 a yard; how many yards did he receive?

23. A grocer exchanged 6 firkins of butter, each weighing 56 pounds, at 25 cents a pound, for 4 boxes of tea, each weighing 36 pounds; what was the tea worth a pound?

24. A merchant exchanged 4 pieces of gros grain silk, each containing 50 yards, at 6 dollars a yard, for beaver cloth, worth 5 dollars a yard; how many pieces, each containing 30 yards, did he obtain?

25. At what price per yard will 6 bales of cloth, each containing 12 pieces of 43 yards each, pay for 60 rolls of carpeting of 70 yards each, at \$2.15 per yard?

INTRODUCTION TO FRACTIONS.

IDEAS OF FRACTIONS.

1. If I divide a square into *two equal parts*, what is *one* part called?

2. How many *halves* in a square? How many halves in anything?

3. If I divide a square into *three equal parts*, what is *one* of the parts called?

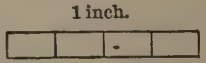
4. What are two of the parts called? How many thirds in a square?

5. Which is larger, one-half or one-third? Two-halves or two-thirds?

6. If I divide a square into *four equal parts*, what is *one* of the parts called?

7. If I divide an inch into four equal parts, what is one part called? What are 2, 3, 4 parts called?

8. How many fourths in the square? How many fourths in half the square?



9. How many fourths in 1 inch? How many fourths in half an inch?

10. To fracture is to break or divide into parts. These equal parts are called **Fractions.** What, then, is a fraction? Name some fractions.

WRITING FRACTIONS.

1. We write 1 *half* thus, $\frac{1}{2}$; we write 1 *third* thus, $\frac{1}{3}$; we write 1 *fourth* thus, $\frac{1}{4}$; 1 *fifth* thus, $\frac{1}{5}$, etc.

2. How shall we write 2 halves? 2 thirds? 2 fourths? 3 fourths? 2 fifths? 1 sixth? 2 sixths? 5 sevenths? 6 eighths?

3. Read the following fractions:

$\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{4}{7}$, $\frac{5}{8}$, $\frac{7}{9}$, $\frac{8}{10}$, $\frac{6}{11}$, $\frac{5}{12}$, $\frac{7}{13}$.

4. The number written below the line is called the *denominator* of the *fraction*, because it gives the *name* to the parts.

5. The number written above the line is called the *numerator*, because it *numbers* the parts.

6. Name the numerator and denominator in the following fractions: $\frac{3}{4}$, $\frac{4}{5}$, $\frac{2}{6}$, $\frac{7}{8}$, $\frac{5}{7}$, $\frac{8}{9}$, $\frac{11}{12}$, $\frac{12}{13}$.

KINDS OF FRACTIONS.

1. How many thirds in *one* square? In *two* squares? How many 4ths in 1 square? In 2 squares?

2. To get 4 thirds how many squares must we cut into thirds? To get 6 fourths how many squares must we cut into fourths?

3. Which is greater, $\frac{4}{3}$ or *one?* $\frac{5}{4}$ or *one?* $\frac{8}{4}$ or *one?* $\frac{7}{5}$ or *one?* $\frac{9}{6}$ or *one?*

4. Fractions that are greater than a unit are called *improper fractions*, because they were not thought to be properly fractions.

5. Fractions less than a unit are called *proper fractions*, because they are properly a part of a unit.

6. Tell which are proper and which are improper fractions in the following: $\frac{3}{4}$, $\frac{5}{4}$, $\frac{4}{5}$, $\frac{7}{5}$, $\frac{8}{8}$, $\frac{5}{7}$, $\frac{4}{8}$, $\frac{11}{7}$, $\frac{7}{11}$.

7. If we unite, or "mix together," an integer and a fraction, as 4 and $\frac{2}{3}$—thus, $4\frac{2}{3}$—we have what is called a *mixed number*.

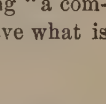
8. Read the following mixed numbers: $2\frac{1}{2}$, $3\frac{2}{3}$, $4\frac{3}{4}$, $3\frac{2}{5}$, $5\frac{3}{6}$, $7\frac{5}{8}$, $8\frac{3}{7}$.

9. If we combine one fraction with another, forming "a compound" of two or more fractions—as, $\frac{1}{2}$ of $\frac{2}{3}$—we have what is called a *compound fraction*.

TREATMENT OF FRACTIONS.

Pupils may be aided in understanding the different cases of fractions by concrete illustrations.

1. What is $\frac{1}{2}$ of $\frac{1}{2}$ of a square?

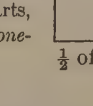


$\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{4}$.

ILLUSTRATION.—To find $\frac{1}{2}$ of $\frac{1}{2}$ we divide one-half into 2 equal parts. Dividing each half into 2 equal parts, there will be *four* parts in all, and each part will be *one-fourth*.

2. What is $\frac{1}{2}$ of $\frac{1}{3}$? $\frac{1}{2}$ of $\frac{1}{4}$? $\frac{1}{2}$ of $\frac{1}{5}$? $\frac{1}{3}$ of $\frac{1}{2}$? $\frac{1}{3}$ of $\frac{1}{4}$? $\frac{1}{3}$ of $\frac{2}{4}$? $\frac{1}{3}$ of $\frac{4}{5}$?

3. How many are $\frac{3}{4}$ of 2?



$\frac{3}{4}$ of $2 = \frac{6}{4}$.

ILLUSTRATION.—Divide each of the figures into fourths, and take 3 fourths of each figure, and we have 6 fourths in all.

4. How many are $\frac{2}{3}$ of 2? $\frac{2}{3}$ of 3? $\frac{3}{4}$ of 3? $\frac{3}{4}$ of 4? $\frac{2}{3}$ of $2\frac{1}{2}$? $\frac{3}{4}$ of $3\frac{2}{3}$?

5. How many thirds in $2\frac{2}{3}$?

ILLUSTRATION.—We see that each unit is 3 thirds, and 2 units are 6 thirds; and these, with the 2 thirds of the third line, make 8 thirds in all.

$2\frac{2}{3} = \frac{8}{3}$

6. How many thirds in $3\frac{1}{3}$? Fourths in $2\frac{3}{4}$? Fifths in $3\frac{4}{5}$? Sixths in $4\frac{5}{6}$?

7. In $\frac{1}{2}$ how many fourths?

ILLUSTRATION.—We see that, dividing the square into 4 equal parts, $\frac{1}{2}$ contains 2 of the parts, or that $\frac{1}{2}$ equals $\frac{2}{4}$.

$\frac{1}{2} = \frac{2}{4}$

8. In $\frac{2}{3}$ how many 6ths? How many 9ths? In $\frac{3}{4}$ how many 8ths? How many 12ths? In $\frac{4}{5}$ how many 10ths?

9. In $\frac{4}{8}$ how many 4ths? How many halves?

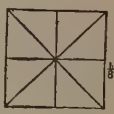


ILLUSTRATION.—Dividing a square into 8 equal parts, we see that 4 of the *eighths* make 2 of the *fourths*, or that $\frac{4}{8} = \frac{2}{4}$.

NOTE.—We can also illustrate with the lines.

$\frac{4}{8} = \frac{2}{4}$

10. In $\frac{8}{12}$ of an inch how many 6ths of an inch? How many 3ds?

11. Reduce $\frac{3}{6}$ to halves; $\frac{4}{6}$ to thirds; $\frac{6}{8}$ to fourths; $\frac{5}{10}$ to halves; $\frac{6}{9}$ to thirds; $\frac{3}{4}$ to 12ths.

12. How many times is $\frac{1}{4}$ contained in $\frac{1}{2}$?

ILLUSTRATION.—In the square it is seen that one-fourth is contained in one-half two times.

13. How many times is $\frac{2}{3}$ contained in $\frac{3}{4}$?

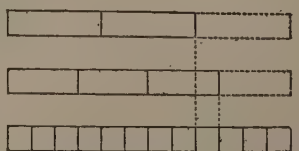


ILLUSTRATION.—We see by the figure that $\frac{2}{3}$ equals $\frac{8}{12}$, and $\frac{3}{4}$ equals $\frac{9}{12}$, and 8 *twelfths* are contained in 9 *twelfths* as often as 8 is contained in 9, which is $\frac{8}{9}$ times, or $1\frac{1}{8}$ times.

SUGGESTION.—Teachers can use these concrete methods in connection with the general treatment of Fractions.

SECTION VII.

COMMON FRACTIONS.

69. One or more of the equal parts of a unit is called a **Fraction**; as one-half, two-thirds, etc.

70. A fraction is expressed by figures with a line between them; thus, $\frac{2}{3}$ expresses 2 *thirds*.

71. The number denoted by the figure below the line is called the **Denominator**; it shows the number of equal parts into which the unit is divided.

Thus, in the fraction $\frac{4}{5}$, 5 is the denominator. It shows that the unit is divided into 5 equal parts.

72. The number denoted by the figure above the line is the **Numerator**; it shows the number of equal parts considered.

Thus, in the fraction $\frac{4}{5}$, 4 is the numerator. It shows that the fraction contains 4 of the 5 equal parts.

73. A fraction whose value is less than a unit is called a **Proper Fraction**; as $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{7}$, etc.

74. A fraction whose value is equal to or greater than a unit is called an **Improper Fraction**; as $\frac{4}{4}$, $\frac{8}{5}$, $\frac{21}{8}$, etc.

75. A number of whole units is called an **Integer**; as 5, 9, 12, etc.

76. An integer and a fraction written together are called a **Mixed Number**; as $2\frac{1}{2}$, $4\frac{2}{3}$, etc.

ORAL EXERCISES.

1. What is one-half?

Ans. One-half of anything is one of the two equal parts of it.

What is

2. One-third?
3. One-fourth?
4. One-fifth?
5. One-sixth?
6. One-seventh?
7. One-eighth?

1. What is two-thirds?

Ans. Two-thirds of anything is two of the three equal parts of it.

What is

2. Two-fourths?
3. Three-fourths?
4. Two-fifths?
5. Three-fifths?
6. Four-fifths?
7. Two-sixths?
8. Three-sevenths?
9. Four-ninths?

1. What is $\frac{1}{2}$ of 6?

Ans. $\frac{1}{2}$ of 6 is 3, since 2 times 3 are 6.

Find

2. $\frac{1}{2}$ of 8.
3. $\frac{1}{3}$ of 12.
4. $\frac{1}{4}$ of 16.
5. $\frac{2}{3}$ of 15.
6. $\frac{3}{4}$ of 20.
7. $\frac{4}{5}$ of 30.
8. $\frac{5}{6}$ of 36.
9. $\frac{6}{7}$ of 42.
10. $\frac{7}{8}$ of 48.

NUMERATION AND NOTATION.

Read the following fractions:

1. $\frac{5}{6}$; $\frac{6}{7}$.
2. $\frac{7}{9}$; $\frac{8}{5}$.
3. $\frac{4}{9}$; $\frac{3}{10}$.
4. $\frac{3}{8}$; $\frac{6}{11}$.
5. $\frac{11}{13}$; $\frac{8}{10}$.
6. $\frac{12}{17}$; $\frac{9}{20}$.
7. $\frac{10}{20}$; $\frac{7}{15}$.
8. $5\frac{3}{7}$; $11\frac{5}{9}$.
9. $6\frac{2}{3}$; $10\frac{4}{5}$.

Write the following fractions:

1. Two-thirds.
2. Four-fifths.
3. Five-sevenths.
4. Six-eighths.
5. Eight-tenths.
6. Seven-ninths.
7. Eleven-fifteenths.
8. Fifteen-sixteenths.

1. Analyze the fraction $\frac{3}{4}$.

SOLUTION.—In the fraction $\frac{3}{4}$ the denominator, 4, shows that the unit is divided into 4 equal parts, and the numerator, 3, shows that 3 of these parts are taken.

Analyze the following:

2. $\frac{2}{3}$.
3. $\frac{5}{6}$.
4. $\frac{3}{7}$.
5. $\frac{4}{7}$.
6. $\frac{4}{5}$.
7. $\frac{2}{11}$.
8. $\frac{4}{8}$.
9. $\frac{7}{9}$.
10. $\frac{12}{13}$.
11. $\frac{3}{11}$.
12. $\frac{8}{11}$.
13. $\frac{8}{14}$.
14. $\frac{9}{15}$.
15. $\frac{18}{21}$.
16. $\frac{21}{31}$.
17. $\frac{7}{18}$.
18. $\frac{18}{23}$.
19. $\frac{34}{44}$.

REDUCTION OF FRACTIONS.

77. The process of changing the form of fractions without changing their value is called **Reduction** of fractions.

78. To reduce mixed numbers to fractions.

ORAL EXERCISES.

1. In $4\frac{2}{3}$ how many thirds?

SOLUTION.—In 1 there are $\frac{3}{3}$, and in 4 there are 4 times $\frac{3}{3}$, or $\frac{12}{3}$, and $\frac{12}{3}$ plus $\frac{2}{3}$ are $\frac{14}{3}$. Therefore, in $4\frac{2}{3}$ there are $\frac{14}{3}$.

2. How many thirds in $4\frac{1}{3}$? In $5\frac{2}{3}$? In $6\frac{1}{3}$? In $7\frac{2}{3}$?

3. How many fourths in $2\frac{1}{4}$? In $4\frac{3}{4}$? In $5\frac{2}{4}$? In $7\frac{3}{4}$?

4. How many fifths in $2\frac{2}{5}$? In $4\frac{3}{5}$? In $6\frac{4}{5}$? In $7\frac{3}{5}$?

5. How many sixths in $2\frac{5}{6}$? In $4\frac{1}{6}$? In $7\frac{3}{6}$? In $8\frac{5}{6}$?

6. How many eighths in $3\frac{1}{8}$? In $4\frac{3}{8}$? In $6\frac{5}{8}$? In $7\frac{7}{8}$?

7. How can we change $4\frac{3}{5}$ to fifths?

8. How, then, may we change a mixed number to a fraction?

WRITTEN EXERCISES.

1. Reduce $6\frac{3}{4}$ to fourths.

SOLUTION.—In 1 there are $\frac{4}{4}$, and in 6 there are 6 times $\frac{4}{4}$, or $\frac{24}{4}$; and $\frac{24}{4}$ plus $\frac{3}{4}$ are $\frac{27}{4}$. From this solution we derive the following method:

OPERATION.

$$6\frac{3}{4}$$
$$4$$
$$\overline{\frac{27}{4}}$$

Multiply the integer by the denominator, add the numerator, and write the denominator under the result.

Reduce to improper fractions—

2. $4\frac{3}{5}$.	**7.** $12\frac{5}{6}$.	**12.** $18\frac{3}{5}$.	**17.** $27\frac{5}{18}$.
3. $7\frac{3}{4}$.	**8.** $13\frac{7}{8}$.	**13.** $21\frac{2}{3}$.	**18.** $51\frac{19}{20}$.
4. $9\frac{5}{6}$.	**9.** $14\frac{5}{9}$.	**14.** $19\frac{11}{12}$.	**19.** $68\frac{21}{25}$.
5. $7\frac{5}{8}$.	**10.** $15\frac{6}{7}$.	**15.** $25\frac{9}{13}$.	**20.** $72\frac{27}{37}$.
6. $8\frac{7}{8}$.	**11.** $17\frac{3}{4}$.	**16.** $35\frac{15}{16}$.	**21.** $89\frac{65}{99}$.

79. To reduce improper fractions to whole or mixed numbers.

80. An improper fraction, being greater than a unit, can be reduced to a whole or mixed number.

ORAL EXERCISES.

1. How many 1's in $\frac{11}{3}$?

SOLUTION.—In 1 there are $\frac{3}{3}$, and in $\frac{11}{3}$ there are as many 1's as 3 is contained times in 11, which is $3\frac{2}{3}$. Therefore in $\frac{11}{3}$ there are $3\frac{2}{3}$.

2. How many ones in $\frac{5}{3}$? In $\frac{7}{4}$? In $\frac{8}{5}$? In $\frac{11}{2}$?
3. How many ones in $\frac{15}{3}$? In $\frac{24}{7}$? In $\frac{18}{6}$? In $\frac{18}{7}$?
4. How many ones in $\frac{16}{5}$? In $\frac{17}{4}$? In $\frac{18}{5}$? In $\frac{17}{7}$?
5. How many ones in $\frac{20}{4}$? In $\frac{21}{7}$? In $\frac{27}{5}$? In $\frac{38}{4}$?
6. How many ones in $\frac{27}{9}$? In $\frac{27}{8}$? In $\frac{27}{7}$? In $\frac{27}{6}$?
7. What did we do with $\frac{11}{3}$ to reduce it to $2\frac{2}{3}$?
8. What did we do with $\frac{15}{3}$ to reduce it to 5?
9. How, then, may we change an improper fraction to a whole or mixed number?

WRITTEN EXERCISES.

1. Reduce $\frac{17}{5}$ to a mixed number.

SOLUTION.—In 1 there are $\frac{5}{5}$, hence in $\frac{17}{5}$ there are as many 1's as 5 is contained times in 17, which are $3\frac{2}{5}$. Hence the following method:

OPERATION.

$$\frac{17}{5} = 17 \div 5 = 3\frac{2}{5}$$

Divide the numerator by the denominator, and the quotient will be the whole or mixed number.

Reduce to whole or mixed numbers—

2. $\frac{9}{4}$.	**7.** $\frac{81}{9}$.	**12.** $\frac{68}{8}$.	**17.** $\frac{215}{15}$.
3. $\frac{11}{8}$.	**8.** $\frac{90}{8}$.	**13.** $\frac{78}{11}$.	**18.** $\frac{216}{18}$.
4. $\frac{19}{5}$.	**9.** $\frac{75}{7}$.	**14.** $\frac{102}{12}$.	**19.** $\frac{324}{24}$.
5. $\frac{32}{4}$.	**10.** $\frac{47}{6}$.	**15.** $\frac{235}{17}$.	**20.** $\frac{440}{40}$.
6. $\frac{72}{6}$.	**11.** $\frac{92}{11}$.	**16.** $\frac{724}{21}$.	**21.** $\frac{550}{50}$.

81. To reduce fractions to higher terms.

82. When a fraction is changed to one having a larger numerator and denominator, it is said to be reduced to **Higher Terms.**

ORAL EXERCISES.

1. How many eighths in $\frac{3}{4}$?

SOLUTION.—In 1 there are $\frac{8}{8}$, and in $\frac{1}{4}$ there are $\frac{1}{4}$ of 8 eighths, or 2 eighths, and in $\frac{3}{4}$ there are 3 times 2 eighths, or 6 eighths.

2. How many fourths in $\frac{1}{2}$? In $\frac{3}{2}$? In $\frac{5}{2}$? In $\frac{7}{2}$?

3. How many sixths in $\frac{2}{3}$? In $\frac{1}{2}$? In $\frac{3}{2}$? In $\frac{5}{3}$?

4. How many eighths in $\frac{1}{2}$? In $\frac{3}{4}$? In $\frac{3}{2}$? In $\frac{6}{4}$?

5. How many ninths in $\frac{2}{3}$? Tenths in $\frac{3}{5}$? In $\frac{7}{5}$?

6. How many twelfths in $\frac{1}{2}$? In $\frac{2}{3}$? In $\frac{3}{4}$? In $\frac{5}{6}$? In $\frac{7}{6}$? In $\frac{5}{4}$?

7. Since $\frac{1}{2}$ equals $\frac{2}{4}$, what may we do to both terms of $\frac{1}{2}$ to change it to $\frac{2}{4}$?

8. Since $\frac{3}{4} = \frac{9}{12}$, what may we do to both terms of $\frac{3}{4}$ to change it to twelfths?

9. How, then, may we reduce a fraction to higher terms?

PRINCIPLE 1.—*Multiplying both numerator and denominator by the same number does not change its value.*

WRITTEN EXERCISES.

1. How many twelfths in $\frac{3}{4}$?

SOLUTION.—Multiplying both numerator and denominator of a fraction by the same number does not change its value, PRIN. 1; hence multiplying both numerator and denominator by 3, we have $\frac{3}{4} = \frac{9}{12}$. From this solution we have the following method:

OPERATION.

$$\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12}.$$

Multiply both numerator and denominator by any number which will give the required denominator.

Reduce

2. $\frac{2}{3}$ to 12ths.	**6.** $\frac{9}{10}$ to 20ths.	**10.** $\frac{8}{9}$ to 36ths.
3. $\frac{5}{6}$ to 30ths.	**7.** $\frac{9}{10}$ to 30ths.	**11.** $\frac{7}{9}$ to 81sts.
4. $\frac{7}{8}$ to 16ths.	**8.** $\frac{4}{15}$ to 30ths.	**12.** $\frac{11}{12}$ to 36ths.
5. $\frac{7}{9}$ to 27ths.	**9.** $\frac{4}{5}$ to 25ths.	**13.** $\frac{14}{15}$ to 60ths.

83. To reduce fractions to lower terms.

84. When a fraction is changed to one having a smaller numerator and denominator, it is reduced to **Lower Terms.**

85. When a fraction cannot be reduced to any lower terms, it is in its **Lowest Terms.**

ORAL EXERCISES.

1. Reduce $\frac{6}{8}$ to fourths.

SOLUTION.—In $\frac{1}{4}$ there are $\frac{2}{4}$, and in $\frac{6}{8}$ there are as many fourths as 2 is contained times in 6, which are 3. Hence $\frac{6}{8}$ equals $\frac{3}{4}$.

2. Reduce $\frac{4}{8}$ to 3ds.	**6.** Reduce $\frac{9}{12}$ to 4ths.
3. Reduce $\frac{6}{9}$ to 3ds.	**7.** Reduce $\frac{10}{12}$ to 6ths.
4. Reduce $\frac{6}{8}$ to 4ths.	**8.** Reduce $\frac{12}{15}$ to 5ths.
5. Reduce $\frac{6}{12}$ to halves.	**9.** Reduce $\frac{16}{20}$ to 5ths.

10. Since $\frac{6}{8}$ equals $\frac{3}{4}$, what may we do to both terms of $\frac{6}{8}$ to obtain the result, $\frac{3}{4}$?

PRINCIPLE 2.—*Dividing both numerator and denominator by the same number does not change the value of the fraction.*

WRITTEN EXERCISES.

1. Reduce $\frac{9}{12}$ to fourths.

SOLUTION.—Dividing both numerator and denominator of a fraction by a number does not change its value, PRIN. 2; hence, dividing both numerator and denominator of $\frac{9}{12}$ by 3, we have $\frac{9}{12}$ equals $\frac{3}{4}$. From this solution we have the following method:

OPERATION.

$$\frac{9}{12} = \frac{9 \div 3}{12 \div 3} = \frac{3}{4}$$

Divide both numerator and denominator by the same number or numbers.

NOTE.—*To reduce to lowest terms, divide until the fraction cannot be reduced any lower.*

Reduce to lowest terms

2. $\frac{6}{9}$.	**7.** $\frac{14}{21}$.	**12.** $\frac{24}{40}$.	**17.** $\frac{27}{45}$.
3. $\frac{8}{12}$.	**8.** $\frac{12}{18}$.	**13.** $\frac{70}{80}$.	**18.** $\frac{84}{96}$.
4. $\frac{10}{12}$.	**9.** $\frac{25}{50}$.	**14.** $\frac{45}{50}$.	**19.** $\frac{108}{120}$.
5. $\frac{16}{24}$.	**10.** $\frac{18}{27}$.	**15.** $\frac{99}{108}$.	**20.** $\frac{121}{132}$.
6. $\frac{16}{28}$.	**11.** $\frac{48}{84}$.	**16.** $\frac{96}{104}$.	**21.** $\frac{144}{156}$.

86. To reduce to a common denominator.

87. When fractions have the same number for a denominator, they are said to have a **Common Denominator.**

88. Fractions having the same number for a denominator are called **Similar Fractions**; as $\frac{2}{4}$ and $\frac{3}{4}$; $\frac{5}{7}$ and $\frac{6}{7}$.

89. Fractions not having the same number for a denominator are called **Dissimilar Fractions**; as $\frac{2}{3}$ and $\frac{5}{6}$.

ORAL EXERCISES.

1. Reduce $\frac{1}{2}$ and $\frac{2}{3}$ to a common denominator.

SOLUTION.—A common denominator for *halves* and *thirds* is *sixths*. In 1 there are $\frac{6}{6}$, and in $\frac{1}{2}$ there are $\frac{1}{2}$ of $\frac{6}{6}$, or $\frac{3}{6}$, etc.

Reduce to a common denominator—

2. $\frac{1}{2}$ and $\frac{3}{4}$.	**5.** $\frac{2}{3}$ and $\frac{3}{5}$.	**8.** $\frac{3}{4}$ and $\frac{3}{5}$.
3. $\frac{2}{3}$ and $\frac{3}{4}$.	**6.** $\frac{2}{3}$ and $\frac{3}{6}$.	**9.** $\frac{3}{4}$ and $\frac{5}{8}$.
4. $\frac{1}{2}$ and $\frac{3}{5}$.	**7.** $\frac{3}{4}$ and $\frac{5}{6}$.	**10.** $\frac{5}{8}$ and $\frac{7}{12}$.

11. Since $\frac{2}{3}$ and $\frac{3}{4}$ equal respectively $\frac{8}{12}$ and $\frac{9}{12}$, what may we do to the terms of $\frac{2}{3}$ and $\frac{3}{4}$ to reduce them to a common denominator?

WRITTEN EXERCISES.

1. Reduce $\frac{3}{4}$ and $\frac{4}{5}$ to a common denominator.

SOLUTION.—Multiplying both numerator and denominator of $\frac{3}{4}$ by 5, the denominator of $\frac{4}{5}$, we have $\frac{3}{4}=\frac{15}{20}$; and multiplying both numerator and denominator of $\frac{4}{5}$ by 4, the denominator of $\frac{3}{4}$, we have $\frac{4}{5}=\frac{16}{20}$; and this makes the fractions have the same denominator; hence the following method:

OPERATION.

$$\frac{3}{4}=\frac{3\times5}{4\times5}=\frac{15}{20}$$

$$\frac{4}{5}=\frac{4\times4}{5\times4}=\frac{16}{20}.$$

Multiply both terms of each fraction by the denominators of the other fractions.

NOTE.—To reduce fractions to their least common denominator *find the least common multiple of the denominators, divide this by the denominator of each fraction, and multiply both terms of the fraction by the quotient.*

Reduce to a common denominator

2. $\frac{2}{3}$ and $\frac{4}{5}$.
3. $\frac{4}{5}$ and $\frac{5}{6}$.
4. $\frac{4}{5}$ and $\frac{9}{10}$.
5. $\frac{7}{8}$ and $\frac{11}{12}$.
6. $\frac{9}{10}$ and $\frac{11}{15}$.
7. $\frac{8}{9}$ and $\frac{9}{10}$.
8. $\frac{5}{12}$ and $\frac{11}{16}$.
9. $\frac{11}{12}$ and $\frac{13}{15}$.
10. $\frac{11}{12}$ and $\frac{17}{18}$.
11. $\frac{13}{15}$ and $\frac{17}{20}$.
12. $\frac{17}{20}$ and $\frac{21}{30}$.
13. $\frac{12}{13}$ and $\frac{13}{14}$.
14. $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$.
15. $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$.
16. $\frac{4}{5}$, $\frac{5}{6}$, and $\frac{9}{10}$.
17. $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{11}{12}$.
18. $\frac{15}{16}$ and $\frac{19}{24}$.
19. $\frac{17}{18}$ and $\frac{25}{36}$.
20. $\frac{23}{20}$ and $\frac{27}{40}$.
21. $\frac{19}{24}$ and $\frac{23}{48}$.
22. $\frac{23}{30}$ and $\frac{31}{60}$.

ADDITION OF FRACTIONS.

90. The process of finding the sum of two or more fractions is called **Addition of Fractions.**

ORAL EXERCISES.

1. How many fourths in $\frac{1}{2}$ and $\frac{3}{4}$?

SOLUTION.—1 *half* equals 2 *fourths;* 2 *fourths* and 3 *fourths* are $\frac{5}{4}$, or $1\frac{1}{4}$.

2. How many fifths in $\frac{3}{5}$ and $\frac{4}{5}$? Eighths in $\frac{6}{8}$ and $\frac{7}{8}$?
3. How many sixths in $\frac{4}{6}$ and $\frac{5}{6}$? Ninths in $\frac{5}{9}$ and $\frac{8}{9}$?
4. How many fourths in $\frac{1}{2}$ and $\frac{3}{4}$? Sixths in $\frac{2}{3}$ and $\frac{5}{6}$?

5. How many eighths in $\frac{3}{4}$ and $\frac{5}{8}$? Ninths in $\frac{2}{3}$ and $\frac{7}{9}$?

6. What is the sum of $\frac{3}{4}$ and $\frac{5}{8}$? Of $\frac{2}{3}$ and $\frac{5}{6}$?

7. What is the sum of $\frac{2}{3}$ and $\frac{5}{9}$? Of $\frac{3}{4}$ and $\frac{5}{12}$?

8. What is the sum of $\frac{2}{5}$ and $\frac{9}{10}$? Of $\frac{3}{4}$ and $\frac{5}{6}$?

9. What kind of fractions can be added without changing their denominators?

10. What must be done to dissimilar fractions before they can be added?

11. How, then, shall we add one fraction to another?

12. Mary had $\frac{2}{5}$ of a dollar, and Sarah had $\frac{4}{5}$ of a dollar; how much did they both have?

13. Lucy gave me $\frac{3}{6}$ of a peach, and Fanny gave me $\frac{5}{6}$ of a peach; how much did I receive?

14. George and Susie had each $\frac{7}{8}$ of a pineapple; how much had they together?

15. If I walk $\frac{2}{3}$ of a mile and ride $\frac{4}{6}$ of a mile, how far do I go?

16. A had $\frac{1}{2}$ of a dollar, B had $\frac{3}{4}$ of a dollar, and C had $\frac{7}{8}$ of a dollar; how much had they all?

WRITTEN EXERCISES.

1. What is the sum of $\frac{2}{3}$ and $\frac{3}{4}$?

SOLUTION.—We first reduce the fractions to a common denominator, that they may be similar fractions: $\frac{2}{3}=\frac{8}{12}$; $\frac{3}{4}=\frac{9}{12}$; 8 *twelfths* plus 9 *twelfths* are 17 *twelfths*. Hence $\frac{2}{3}+\frac{3}{4}=\frac{17}{12}$, or $1\frac{5}{12}$. From this we have the following method:

OPERATION.

$$\frac{2}{3}=\frac{8}{12};\ \frac{3}{4}=\frac{9}{12}$$

$$\frac{8}{12}+\frac{9}{12}=\frac{17}{12}=1\frac{5}{12}$$

Reduce the fractions to a common denominator; add the numerators, and place the sum over the common denominator.

1. Reduce each fraction to its lowest terms before reducing to a common denominator, and also the result after addition.

2. To add mixed numbers, add the integers and fractions separately, and then unite their sums.

Find the sum of the following:

2. $\frac{2}{3}$ and $\frac{2}{5}$.
3. $\frac{3}{4}$ and $\frac{4}{5}$.
4. $\frac{3}{4}$ and $\frac{5}{6}$.
5. $\frac{4}{5}$ and $\frac{6}{7}$.
6. $\frac{3}{7}$ and $\frac{5}{8}$.
7. $\frac{2}{3}$ and $\frac{7}{10}$.
8. $\frac{4}{5}$ and $\frac{8}{9}$.
9. $4\frac{2}{3}$ and $5\frac{3}{4}$.
10. $5\frac{1}{2}$ and $6\frac{1}{3}$.
11. $7\frac{2}{3}$ and $8\frac{5}{6}$.
12. $3\frac{1}{4}$ and $6\frac{5}{8}$.
13. $\frac{7}{8}$ and $\frac{6}{7}$.
14. $\frac{8}{9}$ and $\frac{7}{10}$.
15. $\frac{4}{6}$ and $\frac{12}{14}$.
16. $\frac{10}{12}$ and $\frac{18}{20}$.
17. $\frac{12}{21}$ and $\frac{18}{20}$.
18. $\frac{16}{17}$ and $\frac{18}{24}$.
19. $\frac{19}{20}$ and $\frac{21}{22}$.
20. $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$.
21. $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$.
22. $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{9}{10}$.

23. A has $\frac{2}{3}$ of a pie, and B $\frac{3}{4}$ of a pie; how much have they both?

24. B, having $\frac{3}{4}$ of a ton of hay, bought $\frac{4}{5}$ of a ton; how much had he then?

25. Henry owned $\frac{2}{7}$ of a vessel, and bought $\frac{2}{5}$ of the vessel; how much did he then own?

26. A had $7\frac{3}{4}$ dollars, and B gave him $8\frac{2}{3}$ dollars; how many had he then?

NOTE.—Add the 7 and 8, and then add $\frac{3}{4}$ and $\frac{2}{3}$; $8 + 7 = 15$, $\frac{3}{4} + \frac{2}{3} = \frac{9}{12} + \frac{8}{12} = \frac{17}{12} = 1\frac{5}{12}$; $15 + 1\frac{5}{12} = 16\frac{5}{12}$. *Ans.*

27. A had $25\frac{2}{5}$ acres of land, and then bought $17\frac{3}{4}$ acres; how many had he then?

28. B had $57\frac{3}{5}$ dollars, and C had $96\frac{5}{6}$ dollars; what was the sum of their money?

29. C sold $96\frac{7}{8}$ yards of cloth, and then had $147\frac{9}{16}$ yards left; how much had he at first?

30. What is the distance around a schoolroom $23\frac{2}{3}$ feet long and $18\frac{3}{4}$ feet wide?

31. A boy walked around a house $40\frac{1}{2}$ feet long and $25\frac{2}{3}$ feet wide; how far did he walk?

32. A man gathered $6\frac{2}{3}$ baskets of peaches from one tree, $8\frac{3}{4}$ baskets from another tree, and $10\frac{4}{5}$ baskets from another tree; how many baskets did he gather from the three trees?

SUBTRACTION OF FRACTIONS.

91. The process of finding the difference of two fractions is called **Subtraction of Fractions.**

ORAL EXERCISES.

1. How many fourths in $\frac{3}{4}$ minus $\frac{1}{2}$?

SOLUTION.—1 *half* equals 2 *fourths*, and 3 *fourths* minus 2 *fourths* equals $\frac{1}{4}$.

2. How many fifths in $\frac{4}{5}$ minus $\frac{2}{5}$? In $\frac{7}{5}$ minus $\frac{3}{5}$?

3. How many sixths in $\frac{5}{6}$ minus $\frac{3}{6}$? In $\frac{7}{6}$ minus $\frac{4}{6}$?

4. How many sixths in $\frac{5}{6}$ minus $\frac{1}{2}$? In $\frac{5}{6}$ minus $\frac{2}{3}$?

5. How many eighths in $\frac{7}{8}$ minus $\frac{1}{2}$? In $\frac{7}{8}$ minus $\frac{3}{4}$?

6. How many twelfths in $\frac{3}{4}$ minus $\frac{2}{6}$? In $\frac{5}{6}$ minus $\frac{1}{2}$?

7. Subtract $\frac{3}{4}$ from $\frac{7}{8}$; $\frac{3}{4}$ from $\frac{5}{6}$; $\frac{5}{6}$ from $\frac{11}{12}$.

8. Subtract $\frac{4}{5}$ from $\frac{9}{10}$; $\frac{1}{2}$ from $\frac{7}{10}$; $\frac{2}{3}$ from $\frac{8}{9}$.

9. What kind of fractions can be subtracted without changing their denominators?

10. What must be done to dissimilar fractions before they can be subtracted?

11. How, then, shall we subtract one fraction from another?

12. Mary had $\frac{7}{8}$ of an apple, and gave away $\frac{3}{8}$ of an apple; what part of an apple had she left?

13. Peter found $\frac{4}{5}$ of a dollar, and spent $\frac{2}{5}$ of a dollar; what part of a dollar had he left?

14. If I buy $\frac{3}{4}$ of a ton of hay and sell $\frac{5}{12}$ of a ton, what part of a ton will I have left?

15. John had $\frac{4}{5}$ of a dollar, and gave Jacob $\frac{7}{10}$ of a dollar; how much money had John left?

16. Mary has $\frac{3}{5}$ of a dollar, Sarah $\frac{4}{5}$ of a dollar, and Jane $\frac{2}{3}$ of a dollar; how much more have Mary and Sarah than Jane?

WRITTEN EXERCISES.

1. What is the difference between $\frac{2}{3}$ and $\frac{4}{5}$?

SOLUTION.—We first reduce the fractions to a common denominator that they may express similar fractions: $\frac{4}{5}=\frac{12}{15}$ and $\frac{2}{3}=\frac{10}{15}$, and 12 *fifteenths* minus 10 *fifteenths* is 2 *fifteenths*. From this solution we have the following method:

OPERATION.

$\frac{2}{3}=\frac{10}{15}$; $\frac{4}{5}=\frac{12}{15}$

$\frac{12}{15}-\frac{10}{15}=\frac{2}{15}$

Reduce the fractions to a common denominator, subtract the numerators, and place the result over the common denominator.

1. Reduce each fraction and also the difference to its lowest terms.

2. To subtract mixed numbers, subtract the integers and fractions separately.

Subtract

2. $\frac{2}{3}$ from $\frac{3}{4}$.

3. $\frac{3}{4}$ from $\frac{5}{6}$.

4. $\frac{3}{5}$ from $\frac{5}{6}$.

5. $\frac{3}{7}$ from $\frac{4}{5}$.

6. $\frac{3}{5}$ from $\frac{5}{8}$.

7. $\frac{6}{7}$ from $\frac{11}{12}$.

8. $\frac{5}{8}$ from $\frac{10}{12}$.

9. $\frac{6}{7}$ from $\frac{7}{8}$.

10. $\frac{8}{9}$ from $1\frac{3}{4}$.

11. $\frac{7}{8}$ from $1\frac{1}{2}$.

12. $\frac{8}{12}$ from $\frac{9}{10}$.

13. $\frac{11}{12}$ from $\frac{14}{15}$.

14. $5\frac{1}{4}$ from $8\frac{3}{4}$.

15. $4\frac{3}{7}$ from $8\frac{5}{7}$.

16. $8\frac{5}{9}$ from $12\frac{8}{9}$.

17. $9\frac{6}{11}$ from $15\frac{10}{11}$.

18. $7\frac{7}{12}$ from $16\frac{11}{12}$.

19. $8\frac{5}{13}$ from $21\frac{2}{13}$.

20. Find the difference between $13\frac{1}{6}$ and $7\frac{5}{6}$.

SOLUTION.—Since $\frac{5}{6}$ cannot be subtracted from $\frac{1}{6}$, we take 1 from 13 and add it to the $\frac{1}{6}$, making $12\frac{7}{6}$. Subtracting $7\frac{5}{6}$ from $12\frac{7}{6}$, we have $5\frac{2}{6}$, or $5\frac{1}{3}$.

OPERATION.

$13\frac{1}{6}=12\frac{7}{6}$

$7\frac{5}{6}=7\frac{5}{6}$

$5\frac{2}{6}=5\frac{1}{3}$.

Subtract

21. $5\frac{2}{3}$ from $9\frac{1}{3}$.

22. $7\frac{3}{7}$ from $10\frac{2}{7}$.

23. $5\frac{5}{6}$ from $9\frac{2}{3}$.

24. $6\frac{7}{8}$ from $18\frac{3}{4}$.

25. $9\frac{9}{10}$ from $20\frac{3}{5}$.

26. $18\frac{5}{6}$ from $30\frac{7}{8}$.

27. $25\frac{7}{8}$ from $40\frac{5}{16}$.

28. $36\frac{8}{9}$ from $50\frac{7}{12}$.

29. Mary had $\frac{3}{5}$ of a dollar, and gave away $\frac{1}{4}$ of a dollar; how much had she left?

30. Willie gave Sallie $\frac{4}{5}$ of a quart of peanuts, and Sallie gave him back $\frac{3}{4}$ of a quart; what part of a quart did Sallie keep?

31. A boy has $\frac{7}{8}$ of a pie; if he gives another boy $\frac{2}{8}$ of a pie, how much will remain?

32. A farmer bought $\frac{9}{10}$ of a ton of hay, and sold his neighbor $\frac{3}{7}$ of a ton; how much did the farmer retain?

33. A owned $\frac{1}{4}$ of a vessel; B bought $\frac{1}{5}$ of this, and then sold $\frac{1}{8}$ of what he bought; how much did B keep?

34. The sum of two fractions is $\frac{19}{20}$, and one fraction is $\frac{1}{12}$; what is the other fraction?

35. If a servant-girl had $\frac{4}{5}$ of a certain sum of money, and spent $\frac{3}{4}$ of the sum, how much remained?

36. The sum of three fractions is $\frac{15}{16}$, and two of these fractions are $\frac{1}{3}$ and $\frac{1}{4}$; what is the third fraction?

37. A has $\frac{7}{8}$ of a sum of money; he owes B $\frac{3}{8}$, and C $\frac{1}{8}$ of that sum of money; how much will A have after paying his debts?

PRACTICAL PROBLEMS

(*In Addition and Subtraction of Fractions*).

1. A newsboy earned $17\frac{3}{5}$ dollars, and gave his sister $\$12\frac{4}{5}$; how many dollars remained?

2. Mary has $6\frac{3}{4}$ dollars, and Edith has $7\frac{1}{2}$ dollars; how much have both?

3. A school-boy read $4\frac{1}{2}$ pages one day, and $7\frac{1}{2}$ another day; how many pages did he read in both days?

4. Mary had 20 dollars; she gave her brother $\$9\frac{3}{4}$ and her sister $\$7\frac{2}{5}$; how much remained?

5. William, having \$100, gave $\$26\frac{2}{5}$ to the poor and spent $\$18\frac{3}{4}$ for clothing; how much remained?

6. My father gave me $\$3\frac{3}{5}$, my mother gave me $\$5\frac{3}{4}$, and I then gave my sister $\$4\frac{1}{4}$; how much remained?

7. Maria had \$24, and gave $\frac{1}{3}$ of it to her sister and $\frac{1}{4}$ of it to her brother; how much remained?

8. A type-writer earned \$23, and gave $\$5\frac{3}{4}$ to the poor and $\$11\frac{1}{2}$ for a dress; how much remained?

9. A man had $\$24\frac{3}{4}$, and then earned $\$16\frac{1}{5}$, and then spent $\$10\frac{9}{10}$; how much remained?

10. Peter had $\$17\frac{1}{2}$, and then lost $\$11\frac{1}{5}$, and then earned $\$14$; how much had he then?

11. Harold had $\$\frac{4}{5}$, then lost $\$\frac{3}{4}$, and then earned $\$2\frac{1}{2}$; how much had he then?

12. Henry's father gave him $\$16\frac{3}{4}$, and his mother gave him $\$18\frac{1}{2}$; he then spent $\$24$; how much remained?

COMPOUND FRACTIONS.

92. A fraction of a fraction is called a **Compound Fraction**; as $\frac{1}{2}$ of $\frac{2}{3}$.

ORAL EXERCISES.

1. What is $\frac{1}{2}$ of $\frac{1}{3}$?

SOLUTION.—If we divide a line into 3 equal parts, each part is $\frac{1}{3}$. Then, if we divide each third into two equal parts to get $\frac{1}{2}$ of $\frac{1}{3}$, there will be 3 times 2, or 6 parts, in all; hence each part is $\frac{1}{6}$. Therefore, $\frac{1}{2}$ of $\frac{1}{3}$ is $\frac{1}{6}$.

What is

2. $\frac{1}{2}$ of $\frac{1}{4}$?	**7.** $\frac{1}{4}$ of $\frac{1}{3}$?	**12.** $\frac{2}{3}$ of $\frac{1}{5}$?
3. $\frac{1}{2}$ of $\frac{1}{5}$?	**8.** $\frac{1}{4}$ of $\frac{1}{5}$?	**13.** $\frac{3}{4}$ of $\frac{3}{5}$?
4. $\frac{1}{3}$ of $\frac{1}{4}$?	**9.** $\frac{1}{4}$ of $\frac{1}{6}$?	**14.** $\frac{2}{3}$ of $\frac{5}{6}$?
5. $\frac{1}{3}$ of $\frac{1}{5}$?	**10.** $\frac{1}{4}$ of $\frac{1}{7}$?	**15.** $\frac{3}{5}$ of $\frac{4}{6}$?
6. $\frac{1}{3}$ of $\frac{1}{6}$?	**11.** $\frac{1}{5}$ of $\frac{1}{6}$?	**16.** $\frac{4}{5}$ of $\frac{5}{7}$?

17. Since by analysis we find $\frac{2}{3}$ of $\frac{4}{5} = \frac{8}{15}$, what may we do with $\frac{2}{3}$ and $\frac{4}{5}$ to obtain $\frac{8}{15}$?

18. How, then, may we reduce a compound fraction to a simple one?

19. Since the analysis shows that $\frac{1}{2}$ of $\frac{1}{3}$ equals $\frac{1}{6}$, what do we do to $\frac{1}{3}$ to obtain $\frac{1}{2}$ of it? Hence

PRINCIPLE 3.—*Multiplying the denominator of a fraction by an integer divides the fraction by the integer.*

WRITTEN EXERCISES.

1. What is $\frac{2}{3}$ of $\frac{4}{5}$?

SOLUTION.—$\frac{1}{3}$ of $\frac{4}{5} = \frac{4}{15}$, since multiplying the denominator of a fraction by 3 divides the fraction by 3, PRIN. 3; and if $\frac{1}{3}$ of $\frac{4}{5} = \frac{4}{15}$, $\frac{2}{3}$ of $\frac{4}{5}$ equals 2 times $\frac{4}{15}$, which are $\frac{8}{15}$. From this solution we have the following method:

OPERATION.

$$\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5} = \frac{8}{15}$$

Multiply the numerators together and the denominators together.

NOTE.—It is often best to express the multiplication and cancel common factors before multiplying.

What is

2. $\frac{3}{4}$ of $\frac{7}{8}$?	**7.** $\frac{4}{5}$ of $\frac{15}{16}$?	**12.** $\frac{11}{12}$ of $\frac{16}{33}$?
3. $\frac{5}{6}$ of $\frac{7}{9}$?	**8.** $\frac{9}{10}$ of $\frac{15}{27}$?	**13.** $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{7}{8}$?
4. $\frac{4}{7}$ of $\frac{18}{15}$?	**9.** $\frac{11}{12}$ of $\frac{18}{25}$?	**14.** $\frac{4}{7}$ of $\frac{9}{10}$ of $\frac{14}{16}$?
5. $\frac{8}{9}$ of $\frac{11}{12}$?	**10.** $\frac{12}{13}$ of $\frac{26}{27}$?	**15.** $\frac{3}{5}$ of $\frac{7}{8}$ of $\frac{17}{18}$?
6. $\frac{5}{8}$ of $\frac{9}{10}$?	**11.** $\frac{14}{15}$ of $\frac{35}{42}$?	**16.** $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{14}{36}$?

17. A had $\frac{4}{5}$ of a ton of hay, and sold his neighbor $\frac{2}{3}$ of it; how much did he sell?

SOLUTION.—If A had $\frac{4}{5}$ of a ton of hay, and sold his neighbor $\frac{2}{3}$ of it, he sold his neighbor $\frac{2}{3}$ of $\frac{4}{5}$ of a ton, which is $\frac{8}{15}$ of a ton.

OPERATION.

$$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$$

18. A boy picked $\frac{5}{6}$ of a bushel of strawberries, and sold $\frac{2}{3}$ of them; how many did he sell?

19. A man had $\frac{5}{6}$ of a bushel of barley, and sold $\frac{3}{4}$ of it; how much did he sell?

20. A little girl had $\frac{7}{8}$ of a melon, and gave her brother $\frac{4}{7}$ of it; how much did her brother receive?

21. Says Jennie to Kate, My father owns $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of a ship; what part of the ship did he own?

22. Says Kate to Jennie, My father owns $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of the stock of a bank; what part of the stock did he own?

MULTIPLICATION OF FRACTIONS.

93. The process of multiplying when one or both terms are fractions is called **Multiplication of Fractions.**

94. To multiply when the multiplier is an integer.

ORAL EXERCISES.

1. If a reading book costs $\$\frac{2}{5}$, what will 2 reading books cost at the same rate?

2. If a yard of cloth costs $\$\frac{3}{10}$, what will 3 yards of the cloth cost?

3. If base-balls are worth $\$\frac{3}{4}$ apiece, what will 8 base-balls cost?

4. If it requires $\frac{5}{8}$ of a yard of sateen to make one waist, what will it require to make 9 waists?

5. If a lady has jelly-glasses, each holding $\frac{5}{8}$ of a pint, how much will 12 hold?

6. How many are 3 times $\frac{5}{6}$?

SOLUTION.—3 times 5 sixths are $\frac{15}{6}$, which, reducing to lowest terms, equals $\frac{5}{2}$.

How many are—

7. 2 times $\frac{3}{4}$?	**11.** 4 times $\frac{5}{8}$?	**15.** 6 times $\frac{5}{6}$?
8. 2 times $\frac{5}{8}$?	**12.** 4 times $\frac{7}{12}$?	**16.** 6 times $\frac{11}{2}$?
9. 3 times $\frac{4}{6}$?	**13.** 5 times $\frac{9}{10}$?	**17.** 7 times $\frac{13}{14}$?
10. 3 times $\frac{7}{9}$?	**14.** 5 times $\frac{14}{15}$?	**18.** 8 times $\frac{17}{24}$?

19. Since 3 times $\frac{5}{6}$ equals $\frac{15}{6}$, what do we do to the numerator to obtain the result?

20. Since 3 times $\frac{5}{6}$ equals $\frac{5}{2}$, what do we do to the denominator to obtain the result?

PRINCIPLE 4.—*Multiplying the numerator or dividing the denominator of a fraction by an integer multiplies the fraction by the integer.*

WRITTEN EXERCISES.

1. Multiply $\frac{5}{8}$ by 4.

SOLUTION.—4 times $\frac{5}{8}$ equals $\frac{20}{8}$, according to PRIN. 4. Or, 4 times $\frac{5}{8}$ equals $\frac{5}{2}$, since dividing the denominator multiplies the fraction, according to PRIN. 4. From this we have the following:

OPERATION.

$\frac{5}{8} \times 4 = \frac{20}{8}$

or, $\frac{5}{8} \times 4 = \frac{5}{2}$

To multiply a fraction by an integer, multiply the numerator or divide the denominator by the integer.

Multiply

2. $\frac{9}{10}$ by 5.	**6.** $\frac{17}{18}$ by 9.	**10.** $5\frac{5}{8}$ by 12.
3. $\frac{11}{16}$ by 4.	**7.** $\frac{20}{21}$ by 7.	**11.** $12\frac{5}{12}$ by 15.
4. $\frac{14}{15}$ by 7.	**8.** $\frac{25}{72}$ by 12.	**12.** $10\frac{5}{7}$ by 18.
5. $\frac{18}{19}$ by 3.	**9.** $\frac{121}{144}$ by 36.	**13.** $17\frac{7}{18}$ by 20.

14. A has $\frac{17}{18}$ of a ton of hay, and B has 3 times as much; how much have both?

15. Mary had $\frac{11}{12}$ of a bushel of peaches, and Mabel had 4 times as many; how many bushels had Mabel?

95. To multiply when the multiplier is a fraction.

ORAL EXERCISES.

1. If $\frac{1}{3}$ of an apple is divided into 2 equal parts, what part of the apple will each part be?

2. If $\frac{1}{4}$ of a square is divided into 3 equal parts, what part of the square will each part be?

3. If a pound of tea is worth $\$\frac{4}{5}$, what will $\frac{1}{3}$ of a pound be worth?

4. A man owned $\frac{3}{4}$ of an acre of land, and sold $\frac{2}{5}$ of it; what part of an acre did he sell?

5. George had $\$\frac{3}{4}$, and spent $\frac{2}{5}$ of it for peanuts; what part of a dollar did he spend?

6. What is the product of $\frac{5}{6}$ by $\frac{3}{4}$?

SOLUTION.—$\frac{5}{6}$ multiplied by $\frac{1}{4}$ equals $\frac{1}{4}$ of $\frac{5}{6}$, which is $\frac{5}{24}$; and $\frac{5}{6}$ multiplied by $\frac{3}{4}$ equals 3 times $\frac{5}{24}$, which is $\frac{15}{24}$, or $\frac{5}{8}$.

What is the product of—

7. $\frac{1}{3}$ by $\frac{1}{6}$?	11. $\frac{3}{5}$ by $\frac{3}{7}$?	15. $\frac{14}{15}$ by $\frac{6}{7}$?
8. $\frac{3}{5}$ by $\frac{1}{2}$?	12. $\frac{4}{5}$ by $\frac{6}{7}$?	16. $\frac{22}{25}$ by $\frac{10}{11}$?
9. $\frac{3}{5}$ by $\frac{3}{4}$?	13. $\frac{8}{9}$ by $\frac{3}{4}$?	17. $\frac{13}{14}$ by $\frac{7}{10}$?
10. $\frac{4}{5}$ by $\frac{3}{4}$?	14. $\frac{7}{10}$ by $\frac{5}{11}$?	18. $\frac{17}{18}$ by $\frac{9}{17}$?

19. Since $\frac{5}{6}$ multiplied by $\frac{3}{4}$ equals $\frac{15}{24}$, what do we do with the numerator and denominator to obtain the result?

WRITTEN EXERCISES.

1. Multiply $\frac{7}{8}$ by $\frac{5}{6}$.

SOLUTION.—$\frac{7}{8}$ multiplied by $\frac{1}{6}$ equals $\frac{1}{6}$ of $\frac{7}{8}$, which is $\frac{7}{48}$, according to PRIN. 3; and $\frac{7}{8}$ multiplied by $\frac{5}{6}$ equals 5 times $\frac{7}{48}$ or $\frac{35}{48}$. Hence the following method:

OPERATION.

$$\frac{7}{8}\times\frac{5}{6}=\frac{7\times5}{8\times6}=\frac{35}{48}.$$

Multiply the numerators together for the numerator, and the denominators together for the denominator of the product.

NOTE.—Cancel common factors before multiplying.

What is the product of

2. $\frac{7}{8}$ by $\frac{6}{7}$?	11. 25 by $\frac{11}{15}$?	20. $\frac{38}{42}$ by $\frac{35}{57}$?
3. $\frac{8}{9}$ by $\frac{7}{8}$?	12. 24 by $\frac{11}{12}$?	21. $\frac{36}{45}$ by $\frac{15}{18}$?
4. $\frac{9}{10}$ by $\frac{5}{6}$?	13. 42 by $\frac{11}{12}$?	22. $\frac{51}{80}$ by $\frac{16}{17}$?
5. $\frac{11}{12}$ by $\frac{9}{14}$?	14. 52 by $\frac{11}{13}$?	23. $\frac{57}{90}$ by $\frac{18}{19}$?
6. $\frac{18}{20}$ by $\frac{15}{16}$?	15. $22\frac{1}{2}$ by $\frac{14}{15}$?	24. $\frac{77}{84}$ by $\frac{21}{55}$?
7. $\frac{21}{25}$ by $\frac{15}{14}$?	16. $23\frac{1}{3}$ by $\frac{17}{20}$?	25. $\frac{88}{95}$ by $\frac{25}{33}$?
8. $\frac{19}{21}$ by $\frac{14}{38}$?	17. $\frac{30}{21}$ by $\frac{28}{25}$?	26. $\frac{2}{3}$ by $\frac{3}{4}$ of $\frac{5}{6}$?
9. $\frac{20}{21}$ by $\frac{14}{15}$?	18. $\frac{25}{29}$ by $\frac{17}{35}$?	27. $\frac{7}{8}$ of $\frac{4}{5}$ by $\frac{25}{28}$?
10. $\frac{24}{25}$ by $\frac{20}{27}$?	19. $\frac{35}{48}$ by $\frac{24}{25}$?	28. $\frac{5}{6}$ of $\frac{7}{8}$ by $\frac{24}{35}$?

29. A has $\frac{7}{8}$ of a ton of hay, and B has $\frac{6}{7}$ as much, plus $2\frac{3}{5}$ tons; how much has B?

30. William has $\$5\frac{1}{4}$, and Henry has $\frac{2}{3}$ as much; how much has Henry?

31. What are $16\frac{2}{3}$ bushels of apples worth, at $\$\frac{4}{5}$ a bushel?

32. What cost $8\frac{7}{8}$ tons of hay at $\$14\frac{4}{5}$ a ton?

33. A piece of cloth contains $25\frac{1}{2}$ yards, at $\$3\frac{3}{4}$ a yard; what will $\frac{2}{5}$ of it cost?

SPECIAL METHOD WITH MIXED NUMBERS.

1. Multiply 8 by $4\frac{2}{3}$.

SOLUTION.—We multiply 8 by 2 and divide by 3, and have $5\frac{1}{3}$; then multiply by 4 and add the product, 32, to $5\frac{1}{3}$, making $37\frac{1}{3}$. Hence, in a mixed number we multiply first by the fraction, and then by the integer.

OPERATION.

$$\begin{array}{r} 8 \\ \underline{4\frac{2}{3}} \\ 3)16 \\ \underline{5\frac{1}{3}} \\ 32 \\ \hline 37\frac{1}{3} \end{array}$$

Multiply

2. 35 by $3\frac{4}{5}$.	**6.** 85 by $6\frac{4}{7}$.	**10.** $7\frac{3}{10}$ by 42.
3. 56 by $6\frac{5}{8}$.	**7.** 74 by $8\frac{5}{6}$.	**11.** $9\frac{9}{11}$ by 36.
4. 64 by $7\frac{7}{8}$.	**8.** 99 by $7\frac{7}{9}$.	**12.** $12\frac{5}{12}$ by 48.
5. 96 by $5\frac{11}{12}$.	**9.** 75 by $6\frac{5}{9}$.	**13.** $13\frac{11}{18}$ by 36.

DIVISION OF FRACTIONS.

96. The process of dividing when one or both terms are fractional is called **Division of Fractions.**

97. To divide when the dividend is a fraction.

ORAL EXERCISES.

1. If $\frac{4}{5}$ of an apple is divided equally between 2 girls, what part of the apple will each have?

2. If $\frac{3}{4}$ of a section is divided into 3 equal farms, what part of a section will be contained in each farm?

3. If 5 pounds of sugar can be bought for $\$\frac{1}{4}$, what is the price per pound?

4. If $\frac{2}{3}$ of a bushel of walnuts is divided among 6 persons, what part of a bushel does each receive?

5. If a dozen eggs cost $\$\frac{3}{10}$, what do they cost apiece?

6. How many times is 4 contained in $\frac{8}{11}$?

SOLUTION.—4 is contained in $\frac{8}{11}$, $\frac{1}{4}$ of $\frac{8}{11}$ times, or $\frac{2}{11}$ times.

How many times is

7. 3 contained in $\frac{3}{4}$?
8. 5 contained in $\frac{15}{16}$?
9. 6 contained in $\frac{12}{18}$?
10. 7 contained in $\frac{14}{15}$?
11. 3 contained in $\frac{7}{8}$?
12. 5 contained in $\frac{8}{9}$?
13. 6 contained in $\frac{5}{11}$?
14. 8 contained in $\frac{9}{10}$?
15. 4 contained in $8\frac{3}{4}$?
16. 6 contained in $12\frac{3}{5}$?

17. Since $\frac{8}{11} \div 4 = \frac{2}{11}$, what do we do to the numerator to obtain this result?

18. Since $\frac{9}{11} \div 4 = \frac{9}{44}$, what do we do to the denominator to obtain this result?

PRINCIPLE 5.—*Dividing the numerator or multiplying the denominator of a fraction by an integer divides the fraction by the integer.*

WRITTEN EXERCISES.

1. Divide $\frac{8}{9}$ by 4; also $\frac{9}{10}$ by 4.

SOLUTION.—$\frac{8}{9}$ divided by 4 equals $\frac{2}{9}$, according to PRIN. 5. $\frac{9}{10}$ divided by 4 equals $\frac{9}{40}$, according to PRIN. 5. From this we have the following method:

OPERATION.

$$\frac{8}{9} \div 4 = \frac{2}{9}$$

$$\frac{9}{10} \div 4 = \frac{9}{40}$$

Divide the numerator or multiply the denominator by the divisor.

Divide

2. $\frac{9}{10}$ by 3.
3. $\frac{8}{11}$ by 4.
4. $\frac{12}{18}$ by 6.
5. $\frac{16}{17}$ by 4.
6. $\frac{24}{26}$ by 8.
7. $\frac{12}{18}$ by 7.
8. $\frac{16}{17}$ by 5.
9. $\frac{18}{7}$ by 8.
10. $3\frac{1}{4}$ by 9.
11. $5\frac{2}{5}$ by 12.
12. $\frac{12}{17}$ by 8.
13. $\frac{24}{25}$ by 16.
14. $\frac{27}{28}$ by 18.
15. $12\frac{4}{5}$ by 12.
16. $\frac{25}{26}$ by 15.

17. A lady gave $3\frac{1}{5}$ dollars to 6 little girls; how much did each receive?

18. If 8 pounds of flour cost $\frac{4}{5}$ of a dollar, what was the price of the flour per pound?

19. A farmer received $\$3\frac{3}{5}$ for 12 pounds of butter; what was the price per pound?

20. If 14 feet is $\frac{2}{5}$ of the height of a pine tree, what part of the height is 1 foot?

21. A messenger-boy walked $15\frac{3}{4}$ miles in 6 days; what was the average distance per day?

22. If $25\frac{5}{8}$ pounds of flour were divided among 15 poor families, what was the share of each?

98. To divide when the divisor is a fraction.

ORAL EXERCISES.

1. If apples are worth $\$\frac{1}{5}$ a peck, how many pecks can be bought for $\$\frac{4}{5}$?

2. How many lamps, each holding $\frac{1}{6}$ of a gallon, can be filled from a can of oil holding $\frac{5}{6}$ of a gallon?

3. How many aprons can be made from $7\frac{1}{2}$ yards of calico if each apron requires $2\frac{1}{2}$ yards?

4. If a girl receives $\$\frac{3}{4}$ a week spending-money, how long would it take her to receive $6?

5. Among how many children can 3 pounds of candy be divided if each receives $\frac{3}{8}$ of a pound?

6. How many times is $\frac{2}{3}$ contained in $\frac{4}{5}$?

SOLUTION.—$\frac{1}{3}$ is contained in 1, 3 times; hence $\frac{2}{3}$ is contained in 1 $\frac{1}{2}$ of 3 times, or $\frac{3}{2}$ times; and if $\frac{2}{3}$ is contained in 1 $\frac{3}{2}$ times, $\frac{2}{5}$ is contained in $\frac{4}{5}$, $\frac{3}{2}$ of $\frac{4}{5}$ times, or $\frac{12}{10}$ or $\frac{6}{5}$ times.

How many times is

7. $\frac{4}{5}$ contained in $\frac{7}{8}$?
8. $\frac{2}{3}$ contained in $\frac{2}{7}$?
9. $\frac{8}{9}$ contained in $\frac{5}{6}$?
10. $\frac{2}{3}$ contained in 6?
11. $\frac{7}{8}$ contained in 7?
12. $\frac{3}{4}$ contained in 11?
13. $\frac{5}{6}$ contained in 30?
14. $\frac{8}{9}$ contained in $2\frac{2}{3}$?
15. $\frac{7}{8}$ contained in $8\frac{3}{4}$?
16. $\frac{4}{9}$ contained in $9\frac{3}{5}$?

17. Since $\frac{5}{6} \div \frac{3}{4} = \frac{20}{18}$, by what do we multiply $\frac{5}{6}$, and by what do we divide the product?

WRITTEN EXERCISES.

1. Divide $\frac{7}{8}$ by $\frac{5}{6}$.

SOLUTION.—1 divided by $\frac{5}{6}$, as shown by the analysis, is $\frac{6}{5}$; hence $\frac{7}{8}$ divided by $\frac{5}{6}$ is $\frac{6}{5}$ of $\frac{7}{8}$, or $\frac{7}{8} \times \frac{6}{5}$, which equals $\frac{42}{40}$, or $\frac{21}{20}$. We thus see that the divisor becomes inverted, and derive the following method:

OPERATION.

$$\frac{7}{8} \div \frac{5}{6} = \frac{7}{8} \times \frac{6}{5} = \frac{21}{20}$$

To divide by a fraction, multiply the dividend by the divisor inverted.

NOTE.—Cancel common factors before multiplying.

Divide

2. $\frac{6}{7}$ by $\frac{3}{4}$.
3. $\frac{4}{5}$ by $\frac{2}{3}$.
4. $\frac{7}{8}$ by $\frac{5}{7}$.
5. $\frac{9}{10}$ by $\frac{3}{8}$.
6. $\frac{11}{12}$ by $\frac{8}{9}$.
7. $\frac{10}{11}$ by $\frac{11}{10}$.
8. $\frac{15}{16}$ by $\frac{9}{12}$.
9. $\frac{21}{32}$ by $\frac{14}{18}$.
10. $\frac{20}{21}$ by $\frac{18}{35}$.
11. $\frac{16}{17}$ by $\frac{8}{21}$.
12. $\frac{12}{15}$ by $\frac{16}{35}$.
13. $\frac{32}{36}$ by $\frac{48}{50}$.
14. $\frac{16}{27}$ by $\frac{8}{9}$.
15. $\frac{18}{25}$ by $\frac{9}{10}$.
16. $\frac{28}{45}$ by $\frac{14}{15}$.
17. $\frac{30}{49}$ by $\frac{20}{21}$.
18. $\frac{38}{45}$ by $\frac{19}{25}$.
19. $\frac{48}{50}$ by $\frac{32}{36}$.

20. How much cloth will $\$4\frac{1}{2}$ buy, at $\$\frac{3}{4}$ per yard?

21. How many times may a can holding $1\frac{1}{2}$ gallons be filled from a barrel of kerosene holding $31\frac{1}{2}$ gallons?

22. How many jars of jelly, at $\$\frac{1}{8}$ apiece, can be bought for $\$\frac{3}{4}$?

23. How many dolls, at $\$\frac{3}{8}$ apiece, can be bought for \$6?

24. When oranges are $\$\frac{1}{8}$ a dozen, how many oranges can be bought for $\$3\frac{1}{4}$?

25. Divide $\frac{4}{5}$ of $\frac{7}{8}$ by $\frac{9}{10}$ of $\frac{21}{16}$.

NOTE.—Invert all the factors of the divisor, and cancel.

26. Divide $\frac{7}{9}$ of $\frac{15}{16}$ by $\frac{15}{18}$ of $\frac{21}{32}$.
27. Divide $\frac{4}{5}$ of $\frac{9}{10}$ of $\frac{16}{17}$ by $\frac{12}{25}$ of $\frac{27}{34}$.
28. Divide $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{6}{7}$ of $\frac{7}{12}$ by $\frac{3}{4}$ of $\frac{8}{9}$ of $\frac{15}{16}$.
29. Divide $\frac{7}{10}$ of $\frac{3}{7}$ of $\frac{5}{11}$ of $\frac{4}{9}$ by $\frac{1}{3}$ of $\frac{4}{11}$ of $\frac{9}{14}$.
30. Divide $\frac{3}{8}$ of $\frac{5}{7}$ of $\frac{16}{17}$ of $\frac{3}{5}$ by $\frac{4}{7}$ of $\frac{14}{17}$ of $\frac{11}{12}$.
31. Divide $\frac{4}{9}$ of $\frac{3}{5}$ of $\frac{7}{18}$ of $\frac{4}{25}$ by $\frac{4}{15}$ of $\frac{7}{45}$ of $\frac{2}{11}$.

SPECIAL METHOD WITH MIXED NUMBERS.

1. Divide $54\frac{2}{3}$ by 4.

SOLUTION.—Dividing 54 by 4, we have 13 and a remainder of 2; 2 equals $\frac{6}{3}$, which, added to $\frac{2}{3}$, equals $\frac{8}{3}$; $\frac{8}{3}$ divided by 4, equals $\frac{2}{3}$; hence the quotient is $13\frac{2}{3}$.

OPERATION.

$$4\overline{)54\frac{2}{3}}$$
$$13\frac{2}{3}$$

2. Divide 42 by $3\frac{1}{3}$.

SOLUTION.—We first reduce both numbers to a common denominator. $3\frac{1}{3}$ equals $\frac{10}{3}$, and 42 equals $\frac{126}{3}$; dividing 126 by 10, we find that $\frac{10}{3}$ is contained in $\frac{126}{3}$, $12\frac{6}{10}$ times; hence the quotient is $12\frac{3}{5}$.

OPERATION.

$$\begin{array}{r|l} 3\frac{1}{3} & 42 \\ 3 & 3 \\ \hline 10 & 126 \\ \hline & 12\frac{3}{5} \end{array}$$

Divide

3. $72\frac{1}{2}$ by 5.	**8.** $90\frac{2}{3}$ by 8.	**13.** 75 by $1\frac{1}{2}$.
4. $97\frac{3}{10}$ by 7.	**9.** $83\frac{1}{4}$ by 9.	**14.** 33 by $4\frac{1}{8}$.
5. $87\frac{4}{5}$ by 10.	**10.** $121\frac{3}{7}$ by 10.	**15.** 40 by $6\frac{2}{3}$.
6. $83\frac{3}{5}$ by 11.	**11.** $138\frac{3}{4}$ by 6.	**16.** 500 by $33\frac{1}{3}$.
7. $106\frac{1}{3}$ by 12.	**12.** $162\frac{2}{3}$ by 12.	**17.** 480 by $37\frac{1}{2}$.

18. How many cows, at $\$32\frac{1}{4}$ each, can be bought for $\$193\frac{1}{2}$?

19. How many acres of land, at $\$33\frac{1}{2}$ an acre, can be bought for $\$234\frac{1}{2}$?

20. If a field containing $25\frac{1}{2}$ acres yields $382\frac{1}{2}$ bushels of wheat, what is the average yield per acre?

REDUCTION OF COMPLEX FRACTIONS.

99. A fractional expression whose numerator or denominator, or both, are fractional, is called a **Complex Fraction.**

100. Complex fractions are regarded as expressions of unexecuted division.

Thus, $\frac{2}{3} \div \frac{3}{4}$ may be written $\frac{\frac{2}{3}}{\frac{3}{4}}$; and $4\frac{2}{3} \div \frac{6}{7}$ may be written $\frac{4\frac{2}{3}}{\frac{6}{7}}$.

WRITTEN EXERCISES.

1. Reduce $\frac{\frac{2}{3}}{\frac{4}{5}}$ to a simple fraction.

SOLUTION.—This fractional expression means that $\frac{2}{3}$ is to be divided by $\frac{4}{5}$, and, inverting the divisor and multiplying, we have $\frac{2}{3}\times\frac{5}{4}$, which equals $\frac{5}{6}$. Hence the following method:

OPERATION.

$$\frac{\frac{2}{3}}{\frac{4}{5}}=\frac{2}{3}\div\frac{4}{5}=\frac{2}{3}\times\frac{5}{4}=\frac{5}{6}$$

Multiply the numerator of the complex fraction by its denominator inverted.

Find the value of the following:

2. $\frac{\frac{3}{4}}{\frac{5}{6}}$.

3. $\frac{4}{\frac{2}{3}}$.

4. $\frac{4}{1\frac{1}{3}}$.

5. $\frac{3\frac{1}{3}}{2\frac{1}{2}}$.

6. $\frac{18\frac{3}{4}}{2\frac{1}{8}}$.

7. $\frac{37\frac{3}{5}}{15}$.

8. $\frac{23\frac{3}{4}}{3\frac{1}{4}+2\frac{1}{5}}$.

9. $\frac{\frac{5}{6}\text{ of }2\frac{2}{5}}{8}$.

10. $\frac{\frac{2}{3}\text{ of }\frac{3}{4}}{3\frac{1}{3}}$.

11. $\frac{\frac{2}{3}\text{ of }\frac{4}{5}}{\frac{4}{5}\text{ of }\frac{5}{6}}$.

12. $\frac{5\frac{1}{4}}{2+1\frac{1}{2}}$.

13. $\frac{\frac{1}{2}+\frac{1}{3}}{\frac{1}{3}-\frac{1}{4}}$.

14. $\frac{2\frac{1}{2}+1\frac{1}{3}}{2\frac{1}{2}+3\frac{1}{3}}$.

15. $\frac{4\frac{1}{2}+3\frac{1}{3}}{6\frac{7}{8}-5}$.

16. $\frac{1\frac{1}{8}+2\frac{1}{4}}{\frac{3}{8}\div\frac{3}{4}}$.

17. $\frac{3-2\frac{1}{4}}{2\frac{1}{2}\div\frac{5}{8}}$.

MISCELLANEOUS EXAMPLES.

Reduce to improper fractions

1. $6\frac{7}{8}$.

2. $9\frac{5}{7}$.

3. $12\frac{7}{9}$.

4. $13\frac{3}{14}$.

5. $24\frac{24}{25}$.

6. $12\frac{15}{81}$.

7. $72\frac{13}{48}$.

8. $56\frac{7}{15}$.

9. $89\frac{13}{21}$.

10. $47\frac{21}{34}$.

11. $35\frac{17}{18}$.

12. $132\frac{20}{21}$.

13. $345\frac{23}{24}$.

14. $547\frac{21}{25}$.

15. $777\frac{54}{107}$.

Reduce to mixed numbers

16. $\frac{146}{12}$.

17. $\frac{178}{18}$.

18. $\frac{192}{14}$.

19. $\frac{157}{48}$.

20. $\frac{676}{53}$.

21. $\frac{300}{87}$.

22. $\frac{575}{79}$.

23. $\frac{584}{92}$.

24. $\frac{847}{109}$.

25. $\frac{408}{124}$.

26. $\frac{607}{225}$.

27. $\frac{1728}{345}$.

28. $\frac{2008}{483}$.

29. $\frac{8305}{572}$.

30. $\frac{80000}{605}$.

Reduce to lowest terms

31. $\frac{121}{182}$. **36.** $\frac{125}{500}$. **41.** $\frac{540}{840}$.

32. $\frac{132}{144}$. **37.** $\frac{360}{504}$. **42.** $\frac{480}{1728}$.

33. $\frac{144}{180}$. **38.** $\frac{216}{648}$. **43.** $\frac{2420}{4840}$.

34. $\frac{182}{196}$. **39.** $\frac{492}{744}$. **44.** $\frac{1320}{1680}$.

35. $\frac{315}{405}$. **40.** $\frac{505}{707}$. **45.** $\frac{9072}{120096}$.

Reduce to simple fractions

46. $\frac{3}{4}$ of $\frac{6}{7}$. **51.** $\frac{7}{4}$ of $1\frac{1}{14}$. **56.** $\frac{5}{8}$ of $8\frac{4}{5}$.

47. $\frac{5}{8}$ of $\frac{12}{18}$. **52.** $\frac{6}{19}$ of $2\frac{1}{9}$. **57.** $\frac{13}{14}$ of $\frac{16}{39}$.

48. $\frac{8}{9}$ of $\frac{15}{16}$. **53.** $\frac{11}{80}$ of $6\frac{4}{11}$. **58.** $\frac{15}{16}$ of $\frac{24}{25}$.

49. $\frac{2}{5}$ of $4\frac{1}{6}$. **54.** $\frac{11}{13}$ of $\frac{78}{55}$. **59.** $7\frac{17}{18}$ of $\frac{27}{35}$.

50. $\frac{3}{7}$ of $8\frac{2}{5}$. **55.** $\frac{30}{7}$ of $3\frac{1}{9}$. **60.** $12\frac{53}{81}$ of $44\frac{3}{50}$.

Find the value of

61. $\frac{4}{5} + \frac{5}{6}$. **67.** $\frac{2}{3} + \frac{3}{4} + \frac{5}{6}$.

62. $\frac{5}{6} + \frac{7}{8}$. **68.** $\frac{2}{3} + \frac{3}{4} + \frac{5}{8}$.

63. $\frac{3}{10} + \frac{11}{12}$. **69.** $\frac{4}{7} + \frac{9}{10} + \frac{11}{14}$.

64. $4\frac{3}{4} + 5\frac{4}{5}$. **70.** $\frac{11}{12} + \frac{14}{15} + \frac{15}{16}$.

65. $6\frac{5}{6} + 7\frac{7}{8}$. **71.** $\frac{14}{15} + \frac{23}{40} + \frac{77}{88}$.

66. $9\frac{3}{4} + 8\frac{7}{10}$. **72.** $\frac{11}{12} + \frac{15}{16} + \frac{17}{24}$.

Find the value of

73. $\frac{8}{9} - \frac{5}{6}$. **79.** $\frac{5}{6} + \frac{7}{8} - \frac{5}{12}$.

74. $\frac{11}{12} - \frac{3}{8}$. **80.** $\frac{3}{8} + \frac{5}{9} - \frac{3}{4}$.

75. $\frac{13}{14} - \frac{13}{21}$. **81.** $\frac{6}{7} + \frac{5}{8} - \frac{7}{12}$.

76. $8\frac{2}{3} - 5\frac{3}{4}$. **82.** $\frac{8}{9} - \frac{7}{8} + \frac{9}{10}$.

77. $9\frac{3}{5} - 6\frac{3}{8}$. **83.** $\frac{10}{11} - \frac{5}{13} - \frac{67}{143}$.

78. $12\frac{7}{9} - 10\frac{5}{6}$. **84.** $\frac{11}{12} - \frac{13}{18} + \frac{19}{36}$.

Find the value of

85. $\frac{3}{4} \times 6$. **91.** $\frac{3}{4} \times \frac{5}{6} \times \frac{8}{9}$.

86. $\frac{5}{6} \times 8$. **92.** $\frac{5}{7} \times \frac{6}{8} \times \frac{14}{15}$.

87. $\frac{7}{8} \times 12$. **93.** $\frac{11}{12} \times \frac{15}{16} \times \frac{18}{22}$.

88. $5\frac{3}{4} \times 8$. **94.** $\frac{16}{21} \times \frac{18}{25} \times \frac{35}{36}$.

89. $7\frac{8}{9} \times 12$. **95.** $\frac{27}{28} \times \frac{49}{54} \times \frac{64}{91}$.

90. $8\frac{5}{9} \times 15$. **96.** $\frac{25}{26} \times \frac{39}{40} \times \frac{56}{75}$.

Find the value of

97. $\frac{6}{7} \div 4$.
98. $\frac{8}{9} \div 6$.
99. $1\frac{2}{3} \div 8$.
100. $24 \div \frac{6}{7}$.
101. $16 \div \frac{8}{9}$.
102. $5\frac{1}{3} \div 12$.
103. $\frac{10}{11} \div \frac{5}{6}$.
104. $\frac{14}{15} \div 1\frac{2}{5}$.
105. $\frac{11}{12} \div 5\frac{1}{2}$.
106. $\frac{14}{15} \div 3\frac{1}{2}$.
107. $6\frac{7}{8} \div 8\frac{1}{4}$.
108. $9\frac{5}{7} \div 12\frac{3}{14}$.

Find the value of

109. $\frac{\frac{3}{4}}{3\frac{3}{4}}$.
110. $\frac{\frac{4}{5}}{4\frac{4}{5}}$.
111. $\frac{\frac{5}{6}}{6\frac{1}{4}}$.
112. $\frac{8\frac{3}{4}}{9\frac{1}{6}}$.
113. $\frac{3\frac{5}{6}}{2\frac{2}{5}}$.
114. $\frac{5\frac{1}{4}}{4\frac{1}{8}}$.
115. $\frac{6\frac{3}{5}}{4\frac{1}{8}}$.
116. $\frac{237}{24\frac{5}{6}}$.
117. $\frac{19\frac{3}{4}}{28\frac{7}{18}}$.
118. $\frac{\frac{1}{2}+\frac{1}{3}}{\frac{1}{3}+\frac{1}{4}}$.
119. $\frac{\frac{1}{3}-\frac{1}{5}}{\frac{1}{4}-\frac{1}{6}}$.
120. $\frac{2\frac{2}{3}}{\frac{4}{5}} + \frac{1\frac{1}{2}}{2\frac{1}{4}}$.
121. $\frac{5\frac{1}{3}}{2\frac{2}{5}} + \frac{6\frac{1}{4}}{4\frac{3}{8}}$.
122. $\frac{4\frac{3}{4}}{7\frac{2}{5}} \div \frac{6\frac{1}{3}}{5\frac{2}{7}}$.
123. $\frac{6\frac{2}{3}}{9\frac{5}{6}} \times \frac{7\frac{3}{4}}{8\frac{4}{5}}$.

PRACTICAL PROBLEMS.

1. What cost 20 apples at $\frac{3}{4}$ of a cent each?
2. What cost 12 quarts of nuts at $8\frac{1}{3}$ cents a quart?
3. What cost 12 bushels of apples at $\$\frac{3}{4}$ a bushel?
4. What cost 35 oranges at $2\frac{2}{5}$ cents apiece?
5. What cost 8 quarts of berries at $18\frac{3}{4}$ cents a quart?
6. What cost 12 yards of ribbon at $12\frac{1}{2}$ cents a yard?
7. What cost 10 bushels of oats at $62\frac{1}{2}$ cents a bushel?
8. What cost $10\frac{2}{3}$ yards of muslin at $8\frac{1}{2}$ cents a yard?
9. What cost $13\frac{1}{2}$ yards of cloth at $\$2\frac{2}{3}$ a yard?
10. What cost $15\frac{1}{3}$ pounds of fish at $9\frac{3}{4}$ cents a pound?
11. What cost $12\frac{4}{5}$ yards of ribbon at $31\frac{1}{4}$ cents a yard?
12. What cost $16\frac{2}{5}$ pounds of raisins at $18\frac{3}{4}$ cents a pound?

13. If one yard of velvet cost \$8, how many yards can be bought for \$57?

14. A boy had $\$7\frac{3}{4}$ and found $\$5\frac{2}{5}$; how much money had he then?

15. A man had $10\frac{1}{2}$ tons of hay, and sold $4\frac{3}{4}$ tons of it; how much had he left?

16. A had $25\frac{2}{3}$ acres of land, and bought $24\frac{1}{4}$ acres; how much land had he then?

17. The sum of two fractions is $\frac{17}{18}$, and one fraction is $\frac{4}{9}$; what is the other fraction?

18. A man earned $\$34\frac{3}{4}$, and spent $\$17\frac{1}{5}$; how much money remained?

19. Thomas had $\$37\frac{1}{2}$, and gave $\$14\frac{3}{4}$ to his sister; how much did he keep?

20. A sold 6 bushels more than $\frac{1}{3}$ of 50 bushels of apples; how many bushels remained?

21. A boy earned $\$25\frac{3}{4}$, and then had $\$45\frac{1}{2}$; how much had he at first?

22. How many bushels of potatoes can be bought for \$25, at $\$\frac{3}{4}$ a bushel?

23. How many pounds of tea, at $\$1\frac{1}{8}$ a pound, can be bought for $\$40\frac{1}{2}$?

24. A man bought $12\frac{1}{2}$ yards of cloth for $\$62\frac{1}{2}$; what did he pay a yard?

25. How many tons of coal, at $\$6\frac{3}{4}$ a ton, can be bought for \$72?

26. If $8\frac{3}{4}$ pounds of grapes cost 49 cents, how much is that a pound?

27. How many yards of cloth, at $\$5\frac{5}{8}$ a yard, can be bought for $\$18\frac{3}{4}$?

28. How many yards of tape, at $6\frac{1}{4}$ cents a yard, can be bought for $58\frac{3}{4}$ cents?

29. How many bushels of wheat, at $\$1\frac{3}{5}$ a bushel, can be bought for $\$242\frac{2}{5}$?

30. How many sheep, at $\$8\frac{3}{4}$ a head, can be bought for $\$157\frac{1}{2}$?

31. A lady bought $25\frac{1}{2}$ yards of muslin for $\$6.24\frac{3}{4}$; what was the price per yard?

32. How much land can be bought for $\$543\frac{3}{4}$, at $\$43\frac{1}{2}$ an acre?

33. A servant-girl bought $15\frac{5}{8}$ pounds of meat for $\$2.18\frac{3}{4}$; what was the price a pound?

34. A man paid \$852 for cows, giving $\$35\frac{1}{2}$ a head; how many did he buy?

35. How many yards of muslin, at $16\frac{2}{3}$ cents a yard, can you buy for $\$2.08\frac{1}{3}$?

36. The product of two fractions is $\frac{3}{8}$, and one fraction is $\frac{14}{15}$; what is the other fraction?

37. Mary had \$25, and spent $\frac{1}{2}$ of it for a dress, and $\frac{1}{2}$ of the remainder for a bonnet; how much then remained?

ARITHMETICAL ANALYSIS.

101. Analysis is the process of solving problems by a comparison of their elements.

In comparing, we reason to the unit and from the unit, the unit being the basis of the reasoning process.

102. To pass from one integer to another.

1. If 6 cows cost \$90, what will 9 cows cost at the same rate?

SOLUTION.—If 6 cows cost \$90, 1 cow cost $\frac{1}{6}$ of \$90, which is \$15, and 9 cows will cost 9 times \$15, which are \$135.

OPERATION.

$$\begin{array}{r} 6)90 \\ \hline 15 \\ \hline 9 \\ \hline 135, \textit{ Ans.} \end{array}$$

2. If 5 hens cost 155 cents, what will 9 hens cost at the same rate?

3. If 6 pigs cost \$42, what will 11 pigs cost at the same rate?

4. If 8 horses cost \$1040, what will 12 horses cost at the same rate?

5. If 7 yards of cloth cost \$35, what will 25 yards cost as the same rate?

6. How much must I pay for 36 cows, at the rate of 7 cows for 196 dollars?

7. What will 16 books cost, at the rate of 8 books for \$10.80?

8. A man bought 68 ducks at the rate of 24 for \$8; what did they cost?

9. If a man can walk 432 miles in 12 days, how far can he walk in 54 days?

10. In 24 years there are 8766 days; how many days are there in 80 years?

11. In 5 square miles there are 3200 acres; how many acres in 75 square miles?

12. If an engine runs 2736 miles in 18 days, how far will it run in 72 days?

103. To pass from a fraction to an integer.

1. If $\frac{3}{4}$ of an acre of land cost \$96, what will 1 acre cost?

SOLUTION.—If $\frac{3}{4}$ of an acre cost \$96, $\frac{1}{4}$ of an acre will cost $\frac{1}{3}$ of \$96, or \$32, and if $\frac{1}{4}$ of an acre cost \$32, $\frac{4}{4}$ of an acre, or 1 acre, will cost 4 times \$32, or \$128.

OPERATION.

$\frac{3}{4}$ acre cost \$96

$\frac{1}{4}$ acre cost \$32

$\frac{4}{4}$ acre cost \$128

2. If $\frac{3}{4}$ of a sum of money is \$84, required the sum.

3. If $\frac{5}{6}$ of the cost of a cow is \$35, required the cost of the cow.

4. What cost 2 boxes of Florida oranges, if $\frac{3}{5}$ of a box cost 6 dollars?

5. What is the distance from New York to Philadelphia, if $\frac{3}{4}$ of the distance is $67\frac{1}{2}$ miles?

6. If the cost of $\frac{5}{6}$ of an acre of land is \$120, what will 5 acres cost at the same rate?

7. If $\frac{6}{7}$ of the cost of a farm is \$7200, what will the whole farm cost at that rate?

8. How much will 12 loads of hay weigh, if $\frac{8}{9}$ of a load weighs 840 pounds?

9. What will 15 horses cost me, if $\frac{3}{7}$ of the price of a horse is 96 dollars?

10. A merchant bought 236 barrels of flour at the rate of \$8 for $\frac{4}{5}$ of a barrel; how much did they cost him?

104. To pass from a unit or fraction to a fraction.

1. If 1 acre of land is worth \$125, what is $\frac{4}{5}$ of an acre worth?

OPERATION.

SOLUTION.—If 1 acre of land is worth \$125, 1 fifth of an acre is worth $\frac{1}{5}$ of \$125, or \$25, and 4 fifths of an acre are worth 4 times \$25, or \$100.

$5\overline{)125}$
25
4
100, *Ans.*

2. If a shipment of flour is worth \$232, what is $\frac{3}{4}$ of the flour worth?

3. A paid \$540 for a pleasure-boat; how much would he have paid if he had given $\frac{5}{6}$ as much?

4. If $\frac{2}{3}$ of a lot of flour cost \$64, what is $\frac{3}{4}$ of the lot worth?

5. If there are 40 pounds in $\frac{2}{3}$ of a bushel of clover-seed, how many pounds are there in $\frac{5}{6}$ of a bushel?

6. If there are 50 pounds in $\frac{5}{6}$ of a bushel of wheat, how many pounds are there in $1\frac{1}{2}$ of a bushel?

7. If there are 49 pounds in $\frac{7}{8}$ of a bushel of rye, how many pounds are there in $\frac{5}{7}$ of a bushel?

8. If there are 147 pounds in $\frac{3}{4}$ of a barrel of flour, how many pounds are there in $\frac{6}{7}$ of a barrel?

9. If there are 154 cubic inches in $\frac{2}{3}$ of a gallon, how many cubic inches in $\frac{5}{7}$ of a gallon?

10. If there are 1536 cubic inches in $\frac{8}{9}$ of a cubic foot, how many cubic inches in $1\frac{1}{2}$ of a cubic foot?

105. Given a fractional part and the remainder, to find the whole.

1. A lady spent $\frac{3}{5}$ of her money, and then had \$36 remaining; how much money had she at first?

SOLUTION.—If she spent $\frac{3}{5}$ of her money, there remained $\frac{5}{5}$ of her money minus $\frac{3}{5}$ of her money, which is $\frac{2}{5}$ of her money, which is \$36. If $\frac{2}{5}$ of her money is \$36, $\frac{1}{5}$ of her money is $\frac{1}{2}$ of \$36, which is \$18, and $\frac{5}{5}$ of her money is 5 times \$18, or \$90.

OPERATION.

$\frac{5}{5} - \frac{3}{5} = \frac{2}{5} = \$36$

$\frac{1}{5} = \$18$

$\frac{5}{5} = \$90$, *Ans.*

2. A man spent $\frac{3}{5}$ of his money, and then had \$60 remaining; how much had he at first?

3. A lady sold $\frac{2}{3}$ of her hens, and then had 60 remaining; how many had she at first?

4. Mrs. Clark sold $\frac{2}{7}$ of her bank-stock, and the remainder was worth \$680; how much had she at first?

5. After giving $\frac{1}{4}$ of her income to the poor, Mrs. Dallas had \$720 remaining; what was her income?

6. A pole stands $\frac{1}{3}$ in the mud and $\frac{1}{4}$ in the water and 15 feet in the air; required the length of the pole.

7. Two-fifths of my money is in bank, $\frac{1}{3}$ in government bonds, and \$480 in cash; what was my money?

8. A sold $\frac{1}{4}$ of his land to B, and $\frac{3}{7}$ to C, and then had 180 acres remaining; how much had he at first?

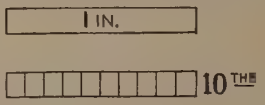
9. One-fourth of a drove of animals are cows, $\frac{1}{5}$ are pigs, and the remainder, 154, are sheep; how many animals are in the drove?

10. A man walked $\frac{3}{7}$ of the distance from Lancaster to Philadelphia one day, $\frac{2}{5}$ of the distance the next day, and the remaining distance, 22 miles, the third day; how far did he walk each day?

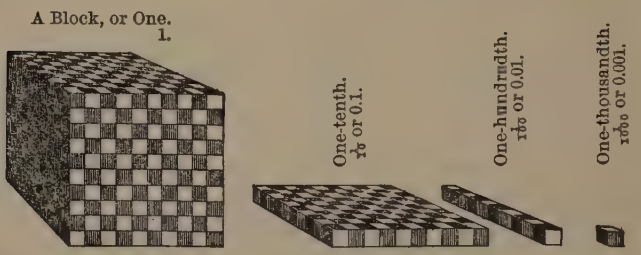
INTRODUCTION TO DECIMALS.

IDEAS OF DECIMALS.

1. If an inch is divided into ten equal parts, what is one of these parts called?



2. How many tenths in one inch? How many tenths in one-half an inch?



3. If a block is divided into 10 equal parts, what is one of the parts called?

4. If one-tenth of the block is divided into 10 equal parts, what is each of these parts called?

5. What, then, is $\frac{1}{10}$ of $\frac{1}{10}$? How many hundredths in $\frac{1}{10}$? In 1?

6. If 1 hundredth of the block is divided into 10 equal parts, what is each part called?

7. What is $\frac{1}{10}$ of $\frac{1}{100}$? How many thousandths in $\frac{1}{100}$? In $\frac{1}{10}$? In 1?

8. These fractions, produced by the successive division by *ten*, are called *decimal fractions*. The word *decimal* is derived from *decem*, meaning *ten*.

WRITING DECIMALS.

1. In 333 the 3 *units* is what part of the 3 *tens*? *Ans. One-tenth.*

2. The 3 *tens* is what part of the 3 *hundreds*? *Ans. One-tenth.*

3. A figure, then, in units' place denotes what part of the value it does in tens' place? *Ans. One-tenth.*

4. We thus see that a figure in any place denotes $\frac{1}{10}$ of what it does in the next place to the left.

5. Suppose that we place a 4 to the *right* of 44, using a period to separate it from the units' place—thus: 44.4; what part of a unit will this 4 denote? *Ans.* 4 *tenths.*

6. In this way how would we express $2\frac{5}{10}$? *Ans.* 2.5.

7. How would we express $8\frac{7}{10}$? $12\frac{4}{10}$? $19\frac{6}{10}$? $24\frac{9}{10}$?

8. In the expression 45.23 the figure 2 expresses only a tenth as much as if it were in units' place, and hence expresses 2 *tenths.*

9. The figure 3 denotes only a tenth as much as if it were in tenths' place, hence it expresses 3 *hundredths.*

10. What shall we name the first place to the right of units? *Ans. Tenths.* The second place to the right? *Ans. Hundredths.* The third place? *Ans. Thousandths.*

11. Write in this way 4 and 2 tenths; 4 and 5 tenths; 4 and 7 tenths; 4 and 3 hundredths; 4 and 6 hundredths; 4 and 8 hundredths.

12. Read the following expressions: 2.3; 7.5; 14.05; 16.25; 32.75; 25.006; 35.205.

13. Read the following expressions: .5; .8; .06; .07; .25; .36; .005; .008; .035; .245.

14. Write the following as decimals: $\frac{3}{10}$; $\frac{5}{10}$; $\frac{25}{100}$; $\frac{46}{100}$; $\frac{7}{100}$; $\frac{8}{100}$; $\frac{125}{1000}$; $\frac{437}{1000}$; $\frac{23}{1000}$; $\frac{9}{1000}$.

NOTE.—In this way we can readily lead the pupil to see how a decimal fraction is written, and the reason for its being thus written.

Another way of teaching pupils to write decimals is not to show the reason for it, but merely to tell them that $\frac{2}{10}$ is written thus, .2; that $\frac{5}{100}$ is written .05, etc. But, taught in this way, they will see no reason why it should be thus written.

SECTION VIII.

DECIMAL FRACTIONS.

106. A number of the decimal divisions of a unit is called a **Decimal Fraction.**

107. A decimal fraction is usually expressed by placing a point before the numerator and omitting the denominator. Thus,

.5 represents $\frac{5}{10}$.	.245 represents $\frac{245}{1000}$.
.25 represents $\frac{25}{100}$.	.006 represents $\frac{6}{1000}$.
.08 represents $\frac{8}{100}$.	5.247 represents $5\frac{247}{1000}$.

108. The point is called the **Decimal Point,** or **Separatrix.** The decimal fraction thus expressed is called a **Decimal.**

109. The method of expressing decimal fractions, which is an extension of the method of notation for integers, is exhibited in the following

NUMERATION AND NOTATION TABLE.

NAMES.	Ten-millions.	Millions.	Hundred-thousands.	Ten-thousands.	Thousands.	Hundreds.	Tens.	Units.	*Decimal point.*	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.	Millionths.	Ten-millionths.
PLACES.	8th,	7th,	6th,	5th,	4th,	3d,	2d,	1st,		2d,	3d,	4th,	5th,	6th,	7th,	8th,
	6	6	6	6	6	6	6	6	.	6	6	6	6	6	6	6

EXERCISES IN NUMERATION.

1. Read the decimal .45.

SOLUTION.—This expresses 4 tenths and 5 hundredths, or 45 hundredths, and may thus be read: 4 tenths 5 hundredths, or 45 hundredths. Hence the following methods:

Begin at tenths, and read each term in order toward the right, giving it its proper denomination.

Or, *Read the decimal as a whole number, and give it the denomination of the last term on the right.*

NOTE.—In reading decimals it is customary to use the word *and* only between an integer and a decimal.

Read the following decimals:

2. .45.	**6.** .046.	**10.** 2.0123.
3. .83.	**7.** .007.	**11.** 4.2057.
4. .126.	**8.** .3216.	**12.** 13.0205.
5. .324.	**9.** .1357.	**13.** 27.0027.

EXERCISES IN NOTATION.

1. Express 25 hundredths in the form of a decimal.

SOLUTION.—25 hundredths equal 2 tenths and 5 hundredths, and this is expressed by writing a decimal point before 25, thus, .25. Hence the following method:

Write the numerator, and then place the decimal point so as to give each term its proper place, prefixing ciphers when necessary.

NOTE.—The word *and* should be used only between integers and decimals, and not between the parts of the decimal.

Express the following in decimal form:

2. Thirty-four hundredths.
3. Seventy-five hundredths.
4. Two tenths six hundredths.
5. Twenty-five thousandths.
6. Four tenths 7 thousandths.
7. Seven tenths 8 thousandths.
8. Five hundred and 25 thousandths.
9. Three tenths 7 ten-thousandths.

10. Thirty and 7 ten-thousandths.

11. Six hundredths 8 millionths.

12. Six hundred and 8 millionths.

13. Four hundred ninety-six millionths.

14. Four hundred and ninety-six millionths.

15. Fifty-six and seventy-eight millionths.

16. Four hundredths, seven ten-thousandths, six hundred thousandths.

17. Nine hundred and sixty-nine hundred thousandths.

18. Thirty-five thousand and eight millionths.

19. Ninety thousand and seven ten-millionths.

20. Eighteen thousand and one hundred-millionth.

ORAL EXERCISES.

1. In 55, the 5 units is what part of the 5 tens?

2. A figure in units place denotes what part of the value it does in tens place?

3. In 5.5, the 5 tenths is what part of the 5 units?

4. A figure in tenths place denotes what part of the value it does in units place?

5. In 5.55, the 5 hundredths is what part of the 5 tenths?

6. A figure in hundredths place denotes what part of the value it does in tenths place?

7. What is the result of annexing a cipher to .5; thus, .50?

8. What is the result of prefixing a cipher to .5; thus, .05?

110. We thus see the truth of the following principles:

1. *Moving the decimal point one place toward the right multiplies the decimal by* 10; *two places, by* 100, *etc.*

2. *Moving the decimal point one place toward the left divides the decimal by* 10; *two places, by* 100, *etc.*

3. *Placing a cipher between the decimal point and a decimal divides the decimal by* 10.

4. *Annexing ciphers to a decimal does not change its value.*

REDUCTION OF DECIMALS.

111. The **Reduction of Decimals** is the process of changing their form without changing their value.

112. There are three cases:

1. To reduce to similar decimals.
2. To reduce decimals to common fractions.
3. To reduce common fractions to decimals.

113. To reduce decimals to similar decimals.

WRITTEN EXERCISES.

1. Reduce .4, .25, and .075 to similar decimals.

SOLUTION.—Since the lowest order of the given decimals is thousandths, all the decimals must be changed to thousandths. Since annexing ciphers to a decimal does not change its value, we have .4 = .400, and .25 = .250. From this solution we derive the following method:

OPERATION.

.4 = .400
.25 = .025
.075 = .075

Annex ciphers to each of the given decimals till it is reduced to the lowest order found in any of the given decimals.

Reduce the following to similar decimals:

2. .6, .38, .057.
3. .23, .026, .0234.
4. .4, .069, .2107.
5. .826, .5715, .00087.
6. .07, .3004, .076, .31826.
7. .514, .362537, .2351.
8. .6, .007871, .03.
9. .85, .0043, .954326.

114. To reduce a decimal to a common fraction.

WRITTEN EXERCISES.

1. Reduce .45 to a common fraction.

SOLUTION.—.45 expressed as a common fraction is $\frac{45}{100}$, which, reduced to its lowest terms, equals $\frac{9}{20}$. Hence the following method:

OPERATION.

$.45 = \frac{45}{100}$
$= \frac{9}{20}$

Write the denominator under the decimal, omit the decimal point, and reduce the common fraction to its lowest terms.

Reduce the following decimals to common fractions:

2. .35.	**6.** .75.	**10.** 8.75.	**14.** .025.
3. .48.	**7.** .80.	**11.** .725.	**15.** .0175.
4. .125.	**8.** .375.	**12.** .075.	**16.** .0625.
5. .625.	**9.** .875.	**13.** .0125.	**17.** .00375.

115. To reduce a common fraction to a decimal.

WRITTEN EXERCISES.

1. Reduce $\frac{3}{4}$ to a decimal.

SOLUTION.—$\frac{3}{4}$ equals $\frac{1}{4}$ of 3. 3 equals 30 tenths; $\frac{1}{4}$ of 30 tenths is 7 tenths and 2 tenths remaining; 2 tenths equals 20 hundredths; $\frac{1}{4}$ of 20 hundredths is 5 hundredths; hence $\frac{3}{4}$ = .75. Hence the following method:

OPERATION.

$\frac{3}{4} = \frac{1}{4}$ of 3 =

4)3.00

.75

1. *Annex ciphers to the numerator, and divide by the denominator.*

2. *Point off as many decimal places in the quotient as there are ciphers annexed.*

When the division is not exact, the remainder may be expressed as a common fraction, or the sign + may be written at the right of the decimal to show that the result is not complete.

Reduce the following common fractions to decimals:

2. $\frac{1}{4}$.	**6.** $\frac{5}{16}$.	**10.** $\frac{13}{25}$.	**14.** $\frac{5}{6}$.
3. $\frac{1}{8}$.	**7.** $\frac{7}{16}$.	**11.** $\frac{19}{25}$.	**15.** $\frac{8}{15}$.
4. $\frac{5}{8}$.	**8.** $\frac{11}{20}$.	**12.** $\frac{21}{32}$.	**16.** $\frac{9}{11}$.
5. $\frac{7}{8}$.	**9.** $\frac{19}{20}$.	**13.** $\frac{27}{32}$.	**17.** $\frac{26}{75}$.

ADDITION OF DECIMALS.

116. The process of finding the sum of two or more decimals is called **Addition of Decimals.**

WRITTEN EXERCISES.

1. What is the sum of 7.5, 18.25, 21.36, and 47.45?

SOLUTION.—We write the numbers so that terms of the same order shall stand in the same column, and begin at the right to add. 5 hundredths, plus 6 hundredths, plus 5 hundredths, equal 16 hundredths, which equal 1 tenth and 6 hundredths; we write the 6 hundredths, and add the 1 tenth to the next sum. 4 tenths, plus 3 tenths, plus 2 tenths, plus 5 tenths, are 14 tenths, and 1 tenth added are 15 tenths, which equal 1 unit and 5 tenths: we write the 5 tenths and add the 1 unit to the sum of the units, etc. Hence the following method:

OPERATION.

7.5
18.25
21.36
47.45
94.56

I. *Write the numbers so that terms of the same order shall stand in the same column.*

II. *Add as in whole numbers, and place the decimal point between the units and the tenths of the sum.*

2. Find the sum of 12.05, 33.24, 47.62, 96.47.

3. Find the sum of 76.24, 89.45, 36.40, 85.75.

4. Find the sum of 79.76, 85.08, 95.42, 237.675.

5. Add 18.79, 147.072, 856.709, 185.8761, 397.05784.

6. Add 59.874, 435.095, 672.328, 976.309, 8467.500843.

7. Add 9 and 7 tenths, 41 and 8 hundredths, 75 and 54 hundredths, 128 and 187 thousandths.

8. Add four and two tenths, 13 and 18 hundredths, 42 and 1 thousandth, and 4 ten-thousandths.

9. Add 146 and 9 tenths, 412 hundred-thousandths, 31 and 416 thousandths, 125 ten-thousandths.

10. Add 34 ten-thousandths, 964 thousandths, 7 and 37 hundredths, 193 and 5 tenths, and 26 and 26 hundredths.

11. Add 76 and 49 hundredths, 127 and 49 thousandths, 496 and 167 thousandths, 985 and 98 ten-thousandths, and 99 and 99 hundred-thousandths.

12. Add 67 and 5 tenths, 125 and 1 thousandth, 5 and 695 hundred-thousandths, 19 and 175 millionths, and 9 and 105 ten-millionths.

13. Four piles of wood contain respectively $6\frac{1}{4}$ cords,

7.316 cords, 11.75 cords, and 15.476 cords; how many cords are there altogether?

14. A bicyclist travelled 45.25 miles the first day, 37.375 the second day, 42.875 miles the third day, 51.5 miles the fourth day, and 47.75 miles the fifth day; how far did he travel in the five days?

SUBTRACTION OF DECIMALS.

117. The process of finding the difference between two decimals is called **Subtraction of Decimals.**

WRITTEN EXERCISES.

1. From 67.35 take 42.63.

SOLUTION.—We write the numbers so that terms of the same order stand in the same column, and begin at the right to subtract. 3 hundredths from 5 hundredths leave 2 hundredths; 6 tenths we cannot subtract from 3 tenths; we therefore take 1 unit from the seven units, which, with three tenths, equal 13 tenths; 6 tenths from 13 tenths leave 7 tenths, etc. Hence the following method:

OPERATION.

67.35
42.63
24.72

I. *Write the subtrahend under the minuend, so that terms of the same order stand in the same column.*

II. *Subtract as in whole numbers, and place the decimal point between the units and tenths of the remainder.*

From

2. 63.72 take 25.81.
3. 96.32 take 73.15.
4. 123.16 take 75.84.
5. 247.125 take 167.183.
6. 9.025 take 6.284.
7. 6.375 take 3.728.
8. 192.294 take 1.0024.
9. 9.327 take 5.0873.

10. From 1 and 1 tenth take 1 tenth and 1 thousandth.

11. From 2 and 2 hundredths take 2 tenths and 2 thousandths.

12. From 3 tenths take 8 ten-thousandths.

13. From 7 take 7 tenths 707 millionths.

14. A cistern contains 235.25 barrels of water, but when full it holds 400 barrels; how much does it lack of being full?

MULTIPLICATION OF DECIMALS.

118. The process of multiplying when one or both terms are decimals is called **Multiplication of Decimals.**

WRITTEN EXERCISES.

1. Multiply 7.23 by .46.

SOLUTION.—Multiplying as in whole numbers, we have 33258; now, if the multiplicand alone were hundredths, the product would be one-hundredth of this, or 332.58; but since the multiplier is also hundredths, the product is one-hundredth of 332.58, which, by moving the decimal point two places to the left becomes .3258. Hence the following method:

OPERATION.

7.23
.46
4338
2892
3.3258

Multiply as in whole numbers, and from the right of the product point off as many decimal places as there are in both multiplier and multiplicand.

When there are not as many figures in the product as there are decimal places in both factors, supply the deficiency by prefixing *ciphers*.

Multiply

2. 15.17 by .18.
3. 26.18 by .25.
4. 53.46 by .35.
5. 67.38 by 1.26.
6. 138.25 by 2.47.
7. 466.72 by 5.29.
8. 407.03 by 7.35.
9. 620.75 by 12.36.
10. 725.82 by 23.08.
11. .00723 by .0317.
12. 1.0309 by .00321.
13. .00567 by .0506.

14. If a man walks 4.75 miles in an hour, how far will he walk in 24.75 hours?

15. Which is larger, and how much, the product or the sum of .5625 and .4375?

16. If an engine pumps 45.25 gallons of water in a minute, how many gallons will it pump in 50.75 minutes?

DIVISION OF DECIMALS.

119. The process of dividing when one or both terms are decimals is called **Division of Decimals.**

WRITTEN EXERCISES.

1. Divide 7.8315 by 2.27.

OPERATION.

$$\begin{array}{r|l} 2.27 & 7.8315 \quad (3.45 \\ & \underline{6\,81} \\ & 1\,021 \\ & \underline{908} \\ & 1135 \\ & \underline{1135} \end{array}$$

SOLUTION.—Dividing as in whole numbers, we obtain a quotient of 345; and since the dividend is the product of the divisor and quotient, the number of decimal places in the dividend must equal the number in the divisor and quotient; hence the number of decimal places in the quotient must equal the number of decimal places in the dividend diminished by the number in the divisor; hence there should be *four* minus *two*, or *two* decimal places in the quotient, therefore the quotient is 3.45. Hence the following method:

Divide as in whole numbers, and point off as many decimal places in the quotient as the number of decimal places in the dividend exceeds the number in the divisor.

1. When there are not so many decimal places in the dividend as in the divisor, annex ciphers before dividing, to make the number of places equal.

2. When the number of figures in the quotient is less than the excess of the decimal places in the dividend over those in the divisor, prefix ciphers to the quotient.

3. When there is a remainder after using all the figures of the dividend, annex ciphers and continue the division.

Divide

2. 25.1328 by 8.
3. 14.1372 by 4.5.
4. 196.1875 by 10.75.
5. 65.9736 by 3.1416.
6. 2450.448 by .5236.
7. 2748.9 by .7854.
8. 18.1771 by 27.13.
9. 127.328 by .07958.
10. 15.90435 by 20.25.
11. 352.0625 by 32.75.

12. If a man walks 4.75 miles per hour, how many hours will he require to walk 64.625 miles?

13. If a barrel of flour cost $4.25, how many barrels can be bought for $47.875?

UNITED STATES MONEY.

120. **United States Money** is expressed in the decimal system.

121. The several denominations are shown in the following table:

10 mills (*m.*) = 1 cent, *ct.* or ¢.	10 dimes = 1 dollar, $.
10 cents = 1 dime, *d.*	10 dollars = 1 eagle, *E.*

100 cents = 1 dollar; 25 cents = $\frac{1}{4}$ dollar; 20 cents = $\frac{1}{5}$ dollar; 50 cents = $\frac{1}{2}$ dollar; 75 cents = $\frac{3}{4}$ dollar; 12$\frac{1}{2}$ cents = $\frac{1}{8}$ dollar.

122. The **Dollar** is the unit; the *dime* is $\frac{1}{10}$ of a dollar, the *cent* $\frac{1}{100}$ of a dollar, the *mill* $\frac{1}{1000}$ of a dollar.

123. The dime is written as tenths, the cent as hundredths, etc., the decimal point being placed between dollars and dimes.

124. Dollars and eagles are read as a number of dollars, and dimes and cents as a number of cents.

The *mill* is merely a nominal value and is not a coin.

EXERCISES IN NUMERATION.

1. Write and read $24.75.

SOLUTION.—The pupil will write this upon the slate or blackboard and say: This is read, 24 dollars, 75 cents.

The pupil will write and read the following:

2. $14.25.	**7.** $50.06.
3. $24.67.	**8.** $48.408.
4. $19.84.	**9.** $96.004.
5. $28.574.	**10.** $105.076.
6. $48.50.	**11.** $976.705.

EXERCISES IN NOTATION.

1. Write six dollars twenty-five cents.

2. Write twenty-five dollars thirty-six cents.

3. Write eight dollars, forty-five cents, six mills.

4. Write twenty dollars, seventy-five cents, two mills.

5. Write six eagles, seven dollars, eighty-four cents.

6. Write four dollars, six dimes, seven cents.

7. Write 25 dollars, five cents, eight mills.

8. Write 35 eagles, 8 dollars, 6 cents, 5 mills.

REDUCTION OF UNITED STATES MONEY.

125. Reduction is the process of changing the denomination without changing the value.

WRITTEN EXERCISES.

1. Reduce 6 dollars to cents.

SOLUTION.—In 1 dollar there are 100 cents; hence, in 6 dollars there are 6 times 100 cents, or 600 cents; or we annex two ciphers.

OPERATION.
$6 = 600 cents.

2. Reduce 2347 cents to dollars.

SOLUTION.—Dividing by 100, or placing the point two places from the right, we find 2347 cents equals $23.47. Hence we derive the following principles:

OPERATION.
2347 cents
= $23.47.

1. *To reduce dollars to cents annex* TWO *ciphers.*

2. *To reduce cents to dollars point off* TWO *figures from the right.*

NOTE.—To reduce a number of dollars and cents to cents, we remove the decimal point; thus, $5.24 = 524 cents.

Reduce

3. $18 to cents.

4. $24 to cents.

5. $385 to cents.

6. $375.60 to cents.

7. 480¢ to dollars.

8. 4800¢ to dollars.

9. 875¢ to dollars.

10. 2835¢ to dollars.

11. $5.47 to cents.

12. $27.05 to cents.

13. 384.5¢ to dollars.

14. 684.5¢ to dollars.

ADDITION OF UNITED STATES MONEY.

126. Addition of United States Money is performed like addition of simple numbers.

WRITTEN EXERCISES.

1. Find the sum of \$24.36, \$96.58, and \$75.42.

SOLUTION.—We write dollars under dollars and cents under cents, and begin at the right to add. 2 and 8 are 10, and 6 are 16 cents, which equals 6 cents and 1 dime; we write the 6 cents under the column of cents, and add the 1 dime to the next column, etc. From this solution we derive the following method:

OPERATION.

\$24.36
96.58
75.42
\$196.36

I. *Write dollars under dollars, cents under cents, etc.*

II. *Add as in simple numbers, and place the point between dollars and cents in the sum.*

2. Add \$48.56, \$39.46, \$24.67, and \$81.09.

3. Add \$23.84, \$97.36, \$52.75, and \$98.27.

4. Add \$73.75, \$48.56, \$39.87, and \$75.48.

5. Add \$46.375, \$97.283, \$72.475, and \$8.396.

6. A farmer bought a cow for \$24.75, a horse for \$150.50, a wagon for \$287.75, and a carriage for \$375.87; how much did he pay for all?

7. A merchant bought flour for \$57.35, some calico for \$96.87, some cloth for \$84.50, some boots for \$52.87, and some muslin for \$75.75; what did they all cost?

8. A tailor sold a coat for \$34.75, a vest for \$8.50, a cloak for \$52.25, a pair of trousers for \$9.75, and some other things for \$28.45; what did he receive for all?

9. A lady bought a table for \$18.25, a looking-glass for \$25.75, a bedstead for \$36.50, a bureau for \$46.25; what did they all cost?

10. A owes \$624.30, B owes \$467.56, C owes \$359.45, D owes \$95.12, E owes \$43.84, F owes \$27.75, G owes \$968.47; H owes \$7.75; required the sum of their debts.

SUBTRACTION OF UNITED STATES MONEY.

127. Subtraction of United States Money is performed like subtraction of simple numbers.

WRITTEN EXERCISES.

1. Subtract \$21.48 from \$46.73.

SOLUTION.—We cannot subtract 8 cents from 3 cents, so we will add 10 cents to 3 cents, making 13 cents; 8 cents from 13 cents leave 5 cents. Now, since we added 10 cents, or 1 dime, to the minuend, we must add 1 dime to the 4 dimes, making 5 dimes: 5 dimes from 7 dimes leaves 2 dimes, etc. Hence the following method:

OPERATION.

\$46.73
27.48
\$19.25

I. *Write dollars under dollars, cents under cents, etc.*

II. *Subtract as in simple numbers, and place the point between dollars and cents in the remainder.*

(2)	(3)	(4)	(5)
\$78.25	\$57.52	\$96.43	\$75.75
13.16	23.28	28.14	23.28

6. From \$129.39 take \$48.91.

7. Find the difference between \$234.16 and \$471.24.

8. A man bought a horse for \$234.50, and sold it for \$228.25; what did he lose?

9. A merchant bought cloth for \$96.75, and sold it for \$110.29; what did he gain?

10. A man bought a farm for \$3640.25, and sold it for \$4000; what did he gain?

11. My house cost \$3480.75, and I sold it for \$4000.50; what did I gain?

12. My horse cost \$240.50, and my carriage cost \$386.25; I sold them for \$680.50; what did I gain?

13. A merchant bought cloth for \$325.50, muslin for \$436.75, and calico for \$625.80; he sold it all for \$1300; how much did he lose?

14. I paid \$4637.25 for a farm, \$3675.25 for building a house, and \$2896.87 for building a barn; I sold my property for \$13000; how much did I gain?

15. I paid \$246.75 for a horse, \$325.45 for a mule, \$42.25 for an ox, \$37.50 for a cow; I sold them all for \$603.50; what was the loss?

MULTIPLICATION OF UNITED STATES MONEY.

128. Multiplication of United States Money is performed like multiplication of simple numbers.

WRITTEN EXERCISES.

1. Multiply \$36.25 by 3.

SOLUTION.—3 times 5 cents are 15 cents, which equal 1 dime and 5 cents; we write the 5 cents, and reserve the 1 dime to add to the next product. 3 times 2 dimes are 6 dimes, and 6 dimes plus 1 dime are 7 dimes, etc. Hence the method is as follows:

OPERATION.

\$36.25
3
\$108.75

Multiply as in simple numbers, and place the decimal point between dollars and cents.

Multiply

2. \$26.14 by 4.
3. \$37.27 by 5.
4. \$48.96 by 7.
5. \$37.52 by 8.
6. \$79.35 by 9.
7. \$48.25 by 12.
8. \$72.27 by 13.
9. \$85.58 by 15.
10. \$92.83 by 32.
11. \$75.32 by 46.

12. If one yard of cloth cost \$3.25, what will 5 yards cost?

13. What will 12 horses cost at the rate of \$150.75 apiece?

14. A man bought 27 oxen at the rate of \$36.25 each; what did they cost?

15. A farmer sold 325 bushels of wheat at \$1.25 a bushel; how much did he receive for it?

16. A miller sold 472 barrels of flour at $4.87 a barrel; how much did he receive for it?

17. A man bought 47 cows for $24.30 each, and sold them for $28.10 each; what was the gain?

18. A drover bought 247 horses for $130.75 each, and sold them for $180.30 each; what did he gain?

19. A farmer bought 327 acres of land at $76.25 an acre, and sold it for $92.50 an acre; what did he gain?

DIVISION OF UNITED STATES MONEY.

129. Division of United States Money is performed like division of simple numbers.

130. To divide a number into equal parts.

WRITTEN EXERCISES.

1. Divide $7.32 into 3 equal parts, or find 1 third of it.

SOLUTION.—1 third of 7 dollars is 2 dollars, and 1 dollar remaining; 1 dollar equals 10 dimes, which, added to 3 dimes, equal 13 dimes. 1 third of 13 dimes equals 4 dimes, and 1 dime remaining, etc. The method is as follows:

OPERATION.

3)$7.32

$2.44, *Ans.*

Divide as in simple numbers, and place the point between dollars and cents.

NOTE.—In business transactions the quotient is not extended beyond cents. It is customary to add 1 cent if the next term is greater than one-half, and neglect it if less than one-half.

2. Divide $7.25 into 5 equal parts.

3. Divide $17.22 into 6 equal parts.

4. Divide $18.48 into 8 equal parts.

5. If 7 pigs cost $36.75, what will 1 pig cost?

6. If 8 cows cost $172.80, what will 1 cow cost?

7. If 3 oxen cost $325.20, what will 5 oxen cost?

8. If 7 hens cost $3.15, what will 12 hens cost?

9. What cost 15 sheep, if 4 sheep cost \$29.24?
10. What cost 25 pounds of butter, if 7 lb. cost \$2.38?
11. What cost 34 acres of land, if 12 acres cost \$50.40?
12. What cost 28 cows, if 35 cows cost \$987?
13. What cost 75 oxen, if 38 oxen cost \$1615?
14. What cost 234 hens, if 75 hens cost \$25.50?

131. To divide one sum of money by another.

WRITTEN EXERCISES.

1. Divide \$736 by \$4.

SOLUTION.—Dividing \$736 by \$4, we have \$184. The method is as follows:

OPERATION.

$$4\overline{)736}\quad 184$$

Reduce both sums to the same denomination, and divide as in simple numbers.

2. Divide \$9600 by \$16.
3. Divide 728¢ by 4¢.
4. Divide \$26325 by \$81.
5. Divide \$75 by 1500¢.
6. Divide 9600¢ by \$16.
7. Divide \$125 by 25 dimes.

8. At 42 dollars each, how many oxen can be bought for \$3276?

9. At \$3.25 apiece, how many pigs can you buy for \$120.25?

10. A boy earned 75 cents a day; how many days did he work to earn \$78.75?

11. A drover paid \$6972 for horses, at \$145.25 apiece; how many did he buy?

12. How many cords of wood can you buy for \$312, at \$3.25 a cord?

13. William earned \$3.25 a day, and paid 75 cents for board; in how many days would he save \$912.50?

14. A merchant paid \$853.25 for a case of silk, including \$1.25, cost of box. How many pieces of silk were in the case if it cost \$53.25 a piece?

BILLS AND ACCOUNTS.

132. A written statement of the quantity and price of articles sold, and the entire value of them, is called a **Bill.**

133. An **Account** is a written statement of the debts and the credits of business transactions.

134. The party who owes is the **Debtor**; the party who is owed is the **Creditor.**

135. A bill is **Receipted** when the words **Received Payment**, or **Paid**, are written at the bottom and signed by the creditor or his agent.

136. The following are a few of the abbreviations in use by business-men:

@,	At.	Do.,	The same.	No.,	Number.
%,	Account.	Doz.,	Dozen.	Pay't,	Payment.
Acc't,	Account.	Dr.,	Debtor.	Per,	By.
Bo't,	Bought.	Hhd.,	Hogshead.	Rec'd,	Received.
Co.,	Company.	Lb.,	Pound.	Yd.,	Yard.
Cr.,	Creditor.	Mdse,	Merchandise.		

137. A bill is made out by the following method:

I. *Find the cost of the several items by multiplying the price of each by the quantity, and take the sum of the several products.*

II. *In an* ACCOUNT *find the difference between the debit and credit amounts.*

138. Make out the following bills:

(1)

POTTSVILLE, *May 8, 1894.*

MR. JOHN WATERS,

Bought of HENRY LANDIS,

8	yds. of muslin,	at	$0.12	$
12	" of cloth,	"	2.75	
15	" of silk,	"	$87\frac{1}{2}$	
		Amount due,		$

(2)

LANCASTER, *April 6, 1895.*

THEO. MILLER,

Bought of DAVID HARRIS,

12	pairs boots,	at $6.75	$	
24	" gaiters,	" 4.25		
30	" slippers,	" 2.25		
28	" rubbers,	" 1.25		
	Amount due,		$	

Received payment,
THEO. MILLER.

(3)

MONTICELLO, *Dec. 17, 1894.*

JOHN J. BROOKS,

Bought of CHARLES HOYT,

36	bbls. of flour,	at $5.35	$	
28	lbs. of beef,	" 0.25		
56	yd. of cloth,	" 2.75		
146	bu. of wheat,	" 0.75		
	Amount due,		$	

Received payment,
CHARLES HOYT.

(4)

NEW YORK, *May 20, 1895.*

JOHN SMITH,

In account with JAS. WILSON,

1895.							
		DR.					
Jan.	1	To 75 lbs. of sugar,	at $0.05	$			
Feb.	5	" 36 yd. of cloth,	" 3.15				
		CR.				$	
Jan.	7	By 75 bu. of corn,	at $0.58				
Feb.	2	" 81 bu. of apples,	" 0.75				
		Balance due,					

(5)

Mr. Henry Knight,

Philadelphia, *April 1, 1895.*

To EDWIN JOHNSON,

1895.		Dr.			
Jan.	4	To 145 bu. wheat,	at $0.75	$	
Jan.	10	" 236 " rye,	" 0.55		
Jan.	20	" 176 " oats,	" 0.45		
1895.		Cr.			$
Jan.	3	By 25 yd. of cloth,	at $3.25		
Jan.	12	" 72 " of silk,	" 2.00		
Feb.	24	" 60 " of cassimere,	" 1.50		
		Balance due,			
		Received payment,			

Edwin Johnson.

BUSINESS PROBLEMS.

Note.—Pupils will put these problems in the form of bills or accounts as above. When no dates are given, date the bills on the day the problems are solved.

1. James Wright bought of Mason & Co., June 12, 1892, 28 lb. of sugar, at 5¢ a pound; 15 pounds of starch, at 10¢ a pound; and 4 boxes of strawberries, at 15¢ a box; what was his whole bill?

2. Mrs. Reed bought of Tyndale & Co., Jan. 16, 1893, the following: 2 doz. stoneware plates, at $1.40 a dozen; 1 set tea-ware, $4.50; 2 doz. glass tumblers, at $4 a dozen; 3 pitchers, at 62½ cents each; required Mrs. Reed's bill.

3. Mr. Peters sold at a country store 225 bu. of oats, at 40¢ a bushel, and 90 bu. of rye, at $0.85 a bushel; he bought 25 yd. of calico, at 8¢ a yard; 35 yd. of muslin, at 11 cents a yard; 10 yd. of cambric, at 10¢ a yard; what is still due Mr. Peters on account?

4. John Wilson bought of Amos Thomas a wagon for $56.50; 2 plows, at $7.50 each; and a wheelbarrow, for $5.25; Wilson sold Thomas 25 bu. of potatoes, at 75¢ a bushel, and 75 bu. of wheat, at 85¢ a bushel; which owes the other, and how much?

SECTION IX.

DENOMINATE NUMBERS.

139. A number used in connection with some particular object is called a **Concrete Number**; as 3 books, 5 horses, etc.

140. The particular object named is called the **Unit** of the number. Thus, in 3 books the unit is *book*.

141. A concrete number in which the unit is fixed by law or custom is called a **Denominate Number**; as, 3 pounds, 5 feet, etc.

142. A denominate number of only one denomination is called a **Simple Denominate Number**; as, 3 feet or 5 quarts.

143. A denominate number of two or more different denominations of the same kind of quantity is called a **Compound Denominate Number.** Thus, 3 feet 2 inches is a compound denominate number.

MEASURES OF WEIGHT.

144. The force with which a body tends to fall to the earth is called its **Weight.**

145. There are several kinds of measures of weight; as, *Troy Weight*, *Avoirdupois Weight*, and *Apothecaries' Weight.*

TROY WEIGHT.

146. **Troy Weight** is used in weighing gold, silver, jewels, etc.

TABLE.

24 grains (gr.)	= 1 pennyweight . . .	pwt.
20 pennyweights . . .	= 1 ounce	oz.
12 ounces	= 1 pound	lb.

ORAL EXERCISES.

1. How many grains in 2 pwt.? In 4 pwt.? In 3 pwt.?

2. How many pwt. in 3 oz.? In 5 oz.? In 48 gr.?

3. How many ounces in 3 lb.? In 5 lb.? In 40 pwt.?

4. How many pounds in 24 oz.? In 60 oz? In 84 oz.?

5. How many grains does a silver dollar weigh? It weighs 412½ grains.

6. How many grains does a gold dollar weigh? It weighs 25.8 grains.

WRITTEN EXERCISES.

1. How many grains in 8 pwt. 12 gr.?

SOLUTION.—1 pennyweight equals 24 grains, and 8 pennyweights equal 8 times 24 grains, which are 192 grains; and 192 grains plus 12 grains equal 204 grains. Therefore, etc.

OPERATION.

1 pwt. = 24 gr.

8 pwt. = 24 gr. × 8 = 192 gr.

192 gr. + 12 gr. = 204 gr.

2. How many pennyweights in 17 oz. 14 pwt.?

3. How many oz. in 16 lb. 10 oz.? In 24 lb. 8 oz.?

4. How many ounces and pwt. in 54 pwt.?

SOLUTION.—There are 20 pennyweights in 1 ounce; hence in 54 pwt. there will be as many ounces as 20 is contained times in 54, which are 2 oz., and 14 pwt. remaining. Therefore, etc.

OPERATION.

20 pwt. = 1 oz.

54 pwt. = (54 ÷ 20) oz.

= 2 oz. 14 pwt.

5. How many pwt. and grains in 275 grains?

6. How many pounds, etc. in 7200 grains?

7. How many pwt. in 5 lb. 6 oz. 10 pwt.?

AVOIRDUPOIS WEIGHT.

147. **Avoirdupois Weight** is used in weighing everything except jewels and the precious metals.

1. The Troy lb. contains 5760 gr.; the Avoirdupois lb., 7000 gr.

2. The Troy oz. contains 480 gr.; the Avoirdupois oz., 437½ gr.

TABLE.

16 ounces . . . . . = 1 pound . . . . . lb.
100 pounds . . . . . = 1 hundredweight . . cwt.
20 hundredweight . . = 1 ton . . . . . T.

A *quarter* means one-fourth of a hundredweight.

ORAL EXERCISES.

1. How many ounces in 3 pounds? In 5 lb.? In 7 lb.?
2. How many pounds in a quarter of flour?
3. How many pounds in "half a hundred" of salt?
4. How many pounds in 32 ounces? In 80 ounces?

WRITTEN EXERCISES.

1. How many ounces in 10 lb. 15 oz.
2. How many pounds in 16 cwt. 75 lb.?
3. How many pounds and ounces in 135 oz.?
4. How many hundredweight in 9 T. 17 cwt.?
5. How many pounds and hundredweight in 724 lb.?
6. How many lb., oz., and cwt. in 1703 oz.?
7. How many T., cwt., and lb. in 48000 oz.?
8. How many ounces does your arithmetic weigh?
9. How many pounds do you weigh? How many ounces?
10. Lift a chair and tell me how many pounds it weighs.

APOTHECARIES' WEIGHT.

148. Apothecaries' Weight is used in prescribing and mixing dry medicines.

TABLE.

20 grains (gr.) . . . . = 1 scruple . . . . . ℈.
3 scruples . . . . . = 1 dram . . . . . ʒ.
8 drams . . . . . = 1 ounce . . . . . ℥.
12 ounces . . . . . = 1 pound . . . . . lb.

ORAL EXERCISES.

1. How many grains in 3 scruples? In 5 scruples? In 1 dram? In 2 drams?

2. How many scruples in 4 drams? In 40 grains?

3. How many drams in 5 oz.? In 12 oz.? In 12 scruples?

4. How many ounces in 3 lb.? In 7 lb.? In 40 dr.?

5. How many pounds in 36 oz.? In 60 oz? In 96 dr.?

WRITTEN EXERCISES.

1. How many grains in 25 ℈? In 7 ℈ and 18 gr?

2. How many scruples in 30 ʒ? In 15 ʒ 1 ℈?

3. How many ounces in 18 ℔? In 16 ℔ 11 ʒ?

4. How many scruples and grains in 277 grains?

5. How many drams and ounces in 310 drams?

6. How many pounds, ounces, and drams in 300 ℈?

MEASURES OF CAPACITY.

149. Measures of Capacity are used to find the quantity of fluids and many dry substances.

150. Measures of capacity are of two kinds: *Liquid Measure* and *Dry Measure.*

LIQUID MEASURE.

151. Liquid Measure is used in measuring nearly all kinds of liquids.

TABLE.

4 gills (gi.) . . . . . = 1 pint . . . . . pt.
2 pint . . . . . = 1 quart . . . . . qt.
4 quarts . . . . . = 1 gallon . . . . . gal.

Give the pupils definite ideas of the *gill*, *pint*, and *quart*. Have them judge of the capacity of cups, tumblers, etc.

ORAL EXERCISES.

1. How many gills in 5 pints? In 7 pints? In 4 quarts? In 1 gallon?

2. How many pints in 12 quarts? In 2 gallons? In 44 gills? In 3 gallons?

3. How many quarts in 6 gallons? In 10 gallons? In 20 pints? In 40 gills?

4. How many gills in a quart? How many pints in a gallon? How many gills in a gallon?

WRITTEN EXERCISES.

1. How many pints in 17 qt. 1 pt.? In 22 qt. 1 pt.?

2. How many quarts in 33 gal. 3 qt.? In 40 gal. 2 qt.?

3. How many gallons in 78 qt.? In 185 pt.?

4. How many gal. in 120 pt.? In 160 pt.? In 257 pt.?

5. How many quarts in 112 pt.? In 144 gi.?

DRY MEASURE.

152. Dry Measure is used in measuring dry substances, as grain, fruit, salt, coal, etc.

TABLE.

2 pints (pt.) . . . . . = 1 quart . . . . . qt.
8 quarts . . . . . = 1 peck . . . . . pk.
4 pecks . . . . . = 1 bushel . . . . . bu.

ORAL EXERCISES.

1. How many pints in 5 quarts? In 9 quarts? In 1 peck? In 1 bushel?

2. How many quarts in 3 pecks? In 7 pecks? In 3 bushels? In 24 pints?

3. How many pecks in 5 bushels? In 10 bushels? In 24 quarts? In 80 pints?

4. How many bushels in 12 pecks? In 32 quarts? In 64 pints? In 64 quarts?

5. How many pints in a peck? How many pints in a bushel? How many quarts in a bushel?

WRITTEN EXERCISES.

1. How many pints in 17 qt. 1 pt.?
2. How many quarts in 1 bu. 3 pk. 5 qt.?
3. How many pecks in 55 quarts? In 96 pints?
4. How many pecks in 145 pints? In 257 pints?
5. How many bushels in 157 quarts? In 203 quarts?
6. How many bushels in 120 pints? In 347 pints?

MEASURES OF EXTENSION.

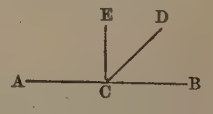
153. Measures of Extension are used to find the *length*, *surface*, and *volume* of bodies.

154. Measures of extension are of three kinds: *Long Measure*, *Surface Measure*, and *Cubic Measure.*

MEASURE OF LENGTH.

155. Measure of Length, or Long Measure, is used for measuring length, breadth, height, distances, etc.

1. A Line is that which has length without breadth or thickness.



2. An **Angle** is the opening between two lines which diverge from a common point. Thus, ACD and DCB are angles.

3. A **Right Angle** is formed by one line perpendicular to another, as ACE or ECB.

TABLE.

12 inches (in.)	= 1 foot	. ft.
3 feet	= 1 yard	. yd.
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ ft. . .	= 1 rod	. rd.
320 rods	= 1 mile	. mi.
3 miles	= 1 league	. l.

NOTE.—In measuring *land* and *roads* we use a *chain*, 4 rods long, divided into 100 *links*. In measuring goods sold by the *yard* we divide the yard into *halves*, *quarters*, *eighths*, etc.

ORAL EXERCISES.

1. How many inches in 3 feet? In 5 feet? In 2 yards?

2. How many feet in 4 yd.? In 2 rd.? In 48 in.?

3. How many yards in 2 rods? In 4 rods? In 36 feet?

4. Draw an inch on the board; draw a foot; draw a yard. Mark off in the schoolroom the length of a rod.

WRITTEN EXERCISES.

1. How many inches in 9 ft. 6 in.? In 12 ft. 8 in.?

2. How many feet in 4 rd. 5 yd.? In 10 rd. 4 yd.?

3. How many yards in 2 miles and 120 rods? In 97 feet?

4. How many feet and inches in 125 in.? In 142 in.?

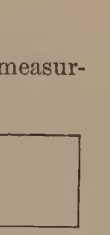
5. How many yards in 72 ft.? In 156 in.? In 275 in.?

6. How many miles and rods in 644 rd.? Rods in 75 yd.?

SURFACE OR SQUARE MEASURE.

156. **Surface** or **Square Measure** is used in measuring surfaces, as land, boards, etc.

1. A **Surface** is that which has length and breadth without thickness.



2. A **Square** is a surface which has four equal sides and four angles, as in the margin.

3. A **Rectangle** is a surface which has four sides and four right angles. A slate, a door, the sides of a room, etc. are examples of rectangles.

4. The **Area** of a surface is expressed by the number of times it contains a small square as a *unit of measure.*

5. *The area of a square or rectangle is equal to the length multiplied by the breadth.* For, in the rectangle above, the whole number of little squares is equal to the number in each row multiplied by the number of rows: that is, 4×3, which equals 12, which is the same as the number of units in length multiplied by the number in breadth.

TABLE.

144 square inches (sq. in.) = 1 square foot . . . sq. ft.
9 square feet . . . . . = 1 square yard . . . sq. yd.
30¼ square yards . . . . = 1 perch, or sq. rod . P.
160 perches . . . . . = 1 acre . . . . . A.
640 acres . . . . . = 1 square mile . . . sq. mi.

NOTE.—In measuring the area of land we have 10,000 square links = 1 square chain, 10 square chains = 1 acre, 640 acres = 1 square mile.

ORAL EXERCISES.

1. Mark off a square inch on the blackboard.

2. Mark off a square foot on the blackboard.

3. Mark off a square yard on the blackboard. Divide it into square feet.

4. How many feet in the length of each side of a square yard? Show how many square feet in a square yard.

5. Divide a square foot into square inches. How many square inches in a square foot?

WRITTEN EXERCISES.

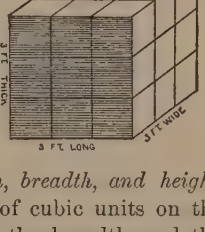
1. How many square inches in 5 square feet?
2. How many square feet in 12 square yards?
3. How many square yards in 8 square rods?
4. In 108 square feet how many square yards?
5. How many square feet in 1728 square inches?
6. How many acres in 960 sq. rd.? In 1250 sq. rd.?
7. How many square rods in 726 sq. yd.?
8. How many acres in 2560 perches?

CUBIC OR SOLID MEASURE.

157. **Cubic or Solid Measure** is used in measuring things which have length, breadth, and thickness.

1. A **Volume** is that which has length, breadth, and thickness. A volume is also called a *solid*.

2. A **Cube** is a volume bounded by six equal squares. A **Rectangular Volume** is one bounded by rectangles. Cellars, boxes, rooms, etc. are examples of rectangular volumes.



3. The **Contents** of a volume are expressed by the number of times it contains a *cube* as a *unit of measure*.

The contents of a cube or rectangular solid are equal to the product of the length, breadth, and height.

For in the volume above, the number of cubic units on the base is equal to the length multiplied by the breadth, and the whole number of cubic units equals the number on the base multiplied by the number of layers: hence the whole number equals $3 \times 3 \times 3 = 27$.

4. A *cord of wood* is a pile 8 feet long, 4 feet wide, and 4 feet high. A *cord foot* is a part of this pile 1 foot long; it equals 16 cubic feet.

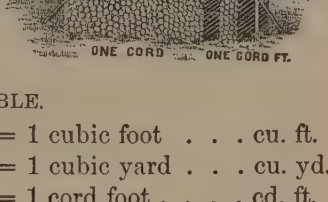


TABLE.

1728 cubic inches (cu. in.) = 1 cubic foot . . . cu. ft.
27 cubic feet . . . . . = 1 cubic yard . . . cu. yd.
16 cubic feet . . . . . = 1 cord foot . . . . cd. ft.
8 cord feet, or 128 cubic feet . . . . = 1 cord of wood . . cd.

NOTE.—Let the teacher take especial pains in explaining this table to the pupils. Show them a cubic inch; draw a cubic foot and a cubic yard on the board. Make a miniature cord of wood out of little sticks, etc. Show also how the contents equal the product of the three dimensions.

WRITTEN EXERCISES.

1. How many cubic feet in 8 cu. yd.? Cubic yards in 540 cu. ft.?

2. How many cubic feet in 6 cords? How many cord feet in 12 cords?

3. How many cubic inches in 7 cu. ft. 96 cu. in.?

4. How many cubic feet in 8469 cu. in.?

MEASURE OF TIME.

158. Time Measure is used for measuring time or duration.

TABLE.

60 seconds (sec.)	= 1 minute . . .	min.
60 minutes	= 1 hour	h. or hr.
24 hours	= 1 day	da.
7 days	= 1 week	wk.
365 days, or 12 months	= 1 common year .	yr.
100 years	= 1 century	C.

WRITTEN EXERCISES.

1. How many seconds in 2 minutes? In 3 minutes? In 4 minutes?

2. How many minutes in 2 hours? In 3 hours? In 4 hours? Hours in 2 days? In 3 days?

3. How many days in 5 wk. 3 da.? In 6 wk. 5 da.? In 3 common years? In 72 hours?

4. How many days in 144 hours? In 216 hours? How many days and hours in 300 hours?

5. How many years in a decade? How many years in a century?

159. For convenience in reckoning time the following divisions of the year are made:

1. The year is divided into 4 seasons—*spring*, *summer*, *autumn*, and *winter*—each consisting of three months. From the following table the pupil may learn the name of each month, the number of days in each, and the months belonging to each season.

2. Every 4 years there are 366 days in a year. The extra day is added to February, making it have 29 days. Such years are called *Leap Years*. Only centennial years divisible by 400 are leap years.

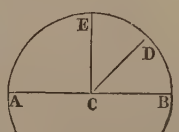
	No. of Month.	Names.	Days.
Spring.	3d month,	March (Mar.),	31.
	4th month,	April (Apr.),	30.
	5th month,	May (May),	31.
Summer.	6th month,	June (June),	30.
	7th month,	July (July),	31.
	8th month,	August (Aug.),	31.
Autumn.	9th month,	September, (Sept.),	30.
	10th month,	October (Oct.),	31.
	11th month,	November (Nov.),	30.
Winter.	12th month,	December (Dec.),	31.
	1st month,	January (Jan.),	31.
	2d month,	February (Feb.),	28; in leap year 29.

The number of days in each month may be easily remembered by means of the following stanza:

Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Excepting February alone,
To which we twenty-eight assign,
Till leap year gives it twenty-nine.

CIRCULAR MEASURE.

160. Circular Measure is used to measure angles and directions, latitude and longitude, etc.



1. A **Circle** is a figure bounded by a curved line, every point of which is equally distant from a point within called the *centre*.

2. The **Circumference** is the bounding line; any part of the circumference, as BD, is an *arc*; AB is the *diameter*, and DC the *radius*.

3. For the purpose of measuring angles the circumference is divided into 360 equal parts, called *degrees;* each degree into 60 equal parts, called *minutes;* each minute into 60 equal parts, called *seconds.*

4. Any angle at the centre, as BCD, is measured by the arc BD included between its sides. A right angle is measured by 90 degrees; half a right angle, by 45 degrees, etc.

TABLE.

60 seconds (″)	= 1 minute	′
60 minutes	= 1 degree	°
30 degrees	= 1 sign	S.
12 signs, or 360°	= 1 circumference	C.

WRITTEN EXERCISES.

1. How many seconds in 4 minutes? In 5′? In 24′?

2. How many minutes in 240 seconds? How many minutes and seconds in 500 seconds?

3. How many minutes in 5 degrees? In 5° 24′? In 16° 36′?

4. How many degrees in 360 minutes? How many degrees and minutes in 650 minutes?

MISCELLANEOUS TABLES.

COUNTING.

12 things = 1 dozen.
12 dozen = 1 gross.
12 gross = 1 great gross.
20 things = 1 score.

TABLE OF PAPER.

24 sheets = 1 quire.
20 quires = 1 ream.
480 sheets = 1 ream.

SIZES OF BOOKS.

A sheet folded in 2 leaves makes a *folio* size.
A sheet folded in 4 leaves " a *quarto*, or *4to*, size.
A sheet folded in 8 leaves " an *octavo*, or *8vo*, size.
A sheet folded in 12 leaves " a *duodecimo*, or *12mo*, size.
A sheet folded in 18 leaves " an *18mo* size.
A sheet folded in 24 leaves " a *24mo* size.

ORAL EXERCISES.

1. How many things in 5 dozen? In 6 dozen? How many dozen in 2 gross? In 5 gross?

2. How many dozen in 60 things? In 72 things? How many gross in 48 dozen? In 97 dozen?

3. How many years in 3 score? Years in "3 score and 10"? Dozen in a great gross?

4. How many sheets in a quire? How many quires in a ream?

WRITTEN EXERCISES.

1. How many pencils in a gross? Pens in a great gross?

2. How many sheets in 4 quires? In 3 quires and 10 sheets?

3. How many quires in 120 sheets? In $4\frac{1}{2}$ reams? In 960 reams?

4. How many sheets in 4 reams? Quires in $6\frac{1}{2}$ reams? Sheets in 3 reams and 4 quires?

REDUCTION OF DENOMINATE NUMBERS.

161. The process of changing the denominations of a denominate number without changing its value is called the **Reduction of Denominate Numbers.**

162. The process of reducing from higher to lower denominations is called **Reduction Descending.**

REDUCTION DESCENDING.

1. Reduce 8 gal. 3 qt. 1 pt. to pints.

SOLUTION.—There are 4 quarts in 1 gallon, and in 8 gallons there are 8 times 4 quarts, or 32 quarts; 32 quarts plus 3 quarts equal 35 quarts; there are 2 pints in 1 quart, and in 35 quarts there are 35 times 2 pints, or 70 pints; 70 pints plus 1 pint, equal 71 pints. Therefore, etc.

OPERATION.

gal.	qt.	pt.
8	3	1
4		
35 qt.		
2		
71 pt.		

WRITTEN EXERCISES.

Reduce to lower denominations

2. 3 oz. 15 pwt. 20 gr.
3. 18 oz. 12 pwt. 18 gr.
4. 35 cwt. 15 lb. 12 oz.
5. 13 T. 10 cwt. 18 lb.
6. 14 ℥ 7 ʒ 2 ℈ 12 gr.
7. 10 ℔ 10 ℥ 15 gr.
8. 16 gal. 3 qt. 1 pt.
9. 15 pk. 7 qt. 1 pt.
10. 10 bu. 6 qt. 1 pt.
11. 5 yd. 2 ft. 6 in.
12. 24 rd. 12 ft. 7 in.
13. 25 gal. 1 qt. 1 pt. 1 gi.
14. 6 bu. 3 pk. 5 qt. 1 pt.
15. 8 mi. 82 rd. 10 ft. 8 in.
16. 5 l. 2 mi. 120 rd. 4 yd.
17. 9 sq. yd. 5 sq. ft.
18. 8 P. 29 sq. yd. 6 sq. ft.
19. 6 A. 100 P. 18 sq. yd.
20. 6 C. 12 cu. ft. 88 cu. in.
21. 7 da. 8 hr. 45 min.
22. 4 wk. 6. da. 12 hr.
23. 7 yr. 240 da. 15 hr.

REDUCTION ASCENDING.

163. The process of reducing a denominate number from lower to higher denominations is called **Reduction Ascending.**

1. Reduce 237 pints to gallons.

SOLUTION.—There are 2 pints in 1 quart, hence in 237 pints there are as many quarts as 2 is contained times in 237, which is 118 qt. and 1 pt. remaining; there are 4 quarts in 1 gallon, hence in 118 qt. there are as many gallons as 4 is contained times in 118, which is 29 gal. and 2 qt. remaining; hence 237 pt. equals 29 gal. 2 qt. 1 pt.

OPERATION.

2 | 237 pt.
4 | 118 qt. — 1 pt.
29 gal. — 2 qt.

237 pt. = 29 gal. 2 qt. 1 pt.

2. Reduce 209 feet to rods.

SOLUTION.—The analysis is similar to that for Ex. 1. In dividing by $5\frac{1}{2}$ it is most convenient to reduce both divisor and dividend to halves before dividing; the remainder will be halves, and must therefore be divided by 2 to give the true remainder.

OPERATION.

3 | 209
$5\frac{1}{2}$ | 69 — 2 ft.
2 | 2
11 | 138
12 rd. — 6 half yd.
= 3 yd.

209 ft. = 12 rd. 3 yd. 2 ft.

WRITTEN EXERCISES.

Reduce

3. 1820 gr. Troy to oz., etc.
4. 3248 gr. Troy to lb., etc.
5. 8946 oz. Av. to lb., etc.
6. 5652 oz. to cwt., etc.
7. 7018 lb. to tons, etc.
8. 1970 oz. to cwt., etc.
9. 2392 gr. Apoth. to oz.
10. 6245 gr. Apoth. to ℔.
11. 1350 pt. to gal., etc.
12. 1630 gi. to gal., etc.
13. 1813 gi. to gal., etc.
14. 1095 pt. to bu., etc.
15. 1443 pt. to bu., etc.
16. 2653 pt. to bu., etc.
17. 4210 in. to yd., etc.
18. 4903 in. to rd., etc.
19. 6948 in. to rd., etc.
20. 8049 ft. to mi., etc.
21. 6315 sq. in. to sq. yd., etc.
22. 8964 sq. ft. to P., etc.
23. 4869 sq. yd. to A., etc.
24. 3020 cu. in. to cu. ft., etc.
25. 9868 cu. ft. to cd., etc.
26. 2528 cu. ft. to cd. ft., etc.
27. 4345 sec. to hr., etc.
28. 4971 min. to da., etc.
29. 8921 hr. to yr., etc.
30. 8730′ to °, etc.
31. 6638 gr. to ℔.
32. 2340 sheets to reams.

ADDITION OF DENOMINATE NUMBERS.

164. Addition of Denominate Numbers is the process of finding the sum of two or more denominate numbers of the same kind of quantity.

WRITTEN EXERCISES.

1. Find the sum of 10 lb. 7 oz. 8 pwt.; 18 lb. 9 oz. 15 pwt.; 16 lb. 11 oz. 12 pwt.

SOLUTION.—We write the numbers so that units of the same kind shall stand in the same column, and begin at the right to add. 12 pwt. plus 15 pwt. plus 8 pwt., equal 35 pwt.; 35 pwt. equal 1 oz. and 15 pwt.; we write the 15 pwt. under the pennyweights, and reserve the 1 oz. to add to the column of ounces. 1 oz. plus 11 oz. plus 9 oz. plus 7 oz., equal 28 oz.; 28 oz. equal 2 lb. 4 oz.; we write the 4 oz. under the ounces column and reserve the 2 lb. to add to the column of pounds, etc.

OPERATION.

lb.	oz.	pwt.
10	7	8
18	9	15
16	11	12
46	4	15

Find the sum of

2. 36 gal. 2 qt. 1 pt.; 42 gal. 1 qt. 1 pt.; 25 gal. 3 qt.

3. 25 pk. 7 qt. 1 pt.; 32 pk. 6 qt. 1 pt.; 24 pk. 4 qt. 1 pt.

4. 15 bu. 3 pk. 6 qt.; 24 bu. 2 pk. 5 qt.; 19 bu. 1 pk. 7 qt.

5. 17 lb. 9 oz. 16 pwt.; 25 lb. 6 oz. 12 pwt.; 72 lb. 11 oz. 13 pwt.; 57 lb. 10 oz. 19 pwt.

6. 18° 19′ 46″; 36° 28′ 21″; 29° 37′ 23″; 42° 41′ 57″.

7. 92 lb. 7 oz. 12 pwt. 23 gr.; 71 lb. 3 oz. 17 pwt. 15 gr.; 28 lb. 9 oz. 10 pwt. 11 gr.; 36 lb. 11 oz. 18 pwt. 15 gr.

8. 17 rd. 4 yd. 2 ft. 6 in.; 21 rd. 2 yd. 1 ft. 7 in.; 23 rd. 3 yd. 8 in.; 25 rd. 5 yd. 2 ft. 9 in.

9. 18 T. 17 cwt. 75 lb. 11 oz.; 12 T. 16 cwt. 66 lb. 12 oz.; 13 T. 17 cwt. 22 lb. 20 oz.; 15 T. 19 cwt. 43 lb. 19 oz.

10. 23 hr. 40 min. 15 sec.; 18 hr. 24 min. 30 sec.; 42 hr. 54 min. 25 sec.; 27 hr. 45 min. 35 sec.

SUBTRACTION OF DENOMINATE NUMBERS.

165. Subtraction of Denominate Numbers is the process of finding the difference between two compound numbers of the same kind of measure.

WRITTEN EXERCISES.

1. From 10 oz. 12 pwt. 20 gr. take 7 oz. 15 pwt. 16 gr.

SOLUTION.—We write the subtrahend under the minuend, writing units of the same name in the same column, and begin at the lowest denomination to subtract. 16 gr. subtracted from 20 gr. leaves 4 gr., which we write under the grains; 15 pwt. from 12 pwt. we cannot take; we therefore take 1 oz. from the 10 oz., leaving 9 oz.; 1 oz. equals 20 pwt., which added to 12 pwt. equal 32 pwt.; 15 pwt. subtracted from 32 pwt. leave 17 pwt., which we write under pwts. 7 oz. from 9 oz. (or, since it will give the same result, we may add 1 oz. to the 7 oz. and say, 8 oz. from 10 oz.) leave 2 oz.

OPERATION.

oz.	pwt.	gr.
10	12	20
7	15	16
2	17	4

2. From 48 gal. 2 qt. 1 pt. take 30 gal. 3 qt. 1 pt.

3. From 16 lb. 10 oz. 16 pwt. take 13 lb. 11 oz. 17 pwt.

4. From 24 bu. 2 pk. 5 qt. take 15 bu. 3 pk. 6 qt. 1 pt.

5. From 80 lb. 8 oz. 14 pwt. 20 gr. take 75 lb. 9 oz. 10 pwt. 23 gr.

6. From 15 T. 12 cwt. 92 lb. 12 oz. take 12 T. 17 cwt. 35 lb. 14 oz.

7. From 42 h. 24 min. 15 sec. take 18 h. 54 min. 30 sec.

8. From 82 mi. 30 rd. 4 yd. 1 ft. take 50 mi. 32 rd. 5 yd. 1 ft.

9. From 216 yr. 10 mo. 2 wk. 5 da. 16 h. take 123 yr. 10 mo. 3 wk. 2 da. 20 h.

MULTIPLICATION OF DENOMINATE NUMBERS.

166. **Multiplication of Denominate Numbers** is the process of multiplying a denominate number by an abstract number.

WRITTEN EXERCISES.

1. Multiply 4 bu. 3 pk. 7 qt. by 9.

SOLUTION.—We write the multiplier under the lowest denomination. 9 times 7 qt. are 63 qt.; 63 qt. = 7 pk. 7 qt.; we write the 7 qt. under the quarts and reserve the 7 pk. to add to the pecks; 9 times 3 pk. are 27 pk.; 27 pk. + 7 pk. = 34 pk.; 34 pk. = 8 bu. and 2 pk.; we write the 2 pk., and reserve the 8 bu. to add to the next product, etc.

OPERATION.

bu.	pk.	qt.
4	3	7
		9
44	2	7

Multiply

2. 16 lb. 8 oz. 15 pwt. by 3

3. 35 gal. 3 qt. 1 pt. by 5.

4. 12 bu. 3 pk. 4 qt. by 4.

5. 18 gal. 3 qt. 1 pt. by 6.

6. 15 lb. 10 oz. 6 pwt. by 8.

7. 14 bu. 6 qt. 1 pt. by 10.

8. 6 yd. 2 ft. 8 in. by 5.

9. 8 yr. 9 mo. 12 da. by 7.

10. 9 wk. 6 da. 14 h. by 12.

11. 7 h. 15 min. 32 sec. by 15.

12. 12 ℔. 8 ℥ 7 ʒ by 16.

13. 13 S. 12° 19′ 32″ by 20.

14. A farmer sold 5 loads of hay, each containing 15 cwt. 3 qr. 15 lb.; how much hay did he sell?

15. If a man walks 16 mi. 240 rd. in each of 21 days, how far will he walk in all?

DIVISION OF DENOMINATE NUMBERS.

167. Division of Denominate Numbers is the process of dividing when one or both terms is a denominate number.

1. Divide 67 lb. 10 oz. 15 pwt. 20 gr. into 5 equal parts; that is, take $\frac{1}{5}$ of it.

SOLUTION.—$\frac{1}{5}$ of 67 lb. is 13 lb., and 2 lb. remaining; 2 lb. equal 24 oz., which added to 10 oz. equal 34 oz.; $\frac{1}{5}$ of 34 oz. is 6 oz., and 4 oz. remaining; 4 oz. equal 80 pwt., which added to 15 pwt. equal 95 pwt.; $\frac{1}{5}$ of 95 pwt. is 19 pwt.; $\frac{1}{5}$ of 20 gr. is 4 gr.

OPERATION.

	lb.	oz.	pwt.	gr.
5)67	10	15	20	
13	6	19	4	

WRITTEN EXERCISES.

Divide

2. 50 lb. 2 oz. 5 pwt. by 3.

3. 176 gal. 3 qt. 1 pt. by 5.

4. 88 bu. 1 pk. 4 qt. by 7.

5. 169 gal. 3 qt. 1 pt. by 9.

6. 34 yd. 1 ft. 4 in. by 5.

7. 126 lb. 10 oz. 8 pwt. by 8.

8. 61 yr. 5 mo. 24 da. by 7.

9. 117 wk. 3 da. 12 hr. by 12.

10. 112 T. 2 cwt. 66 lb. by 7.

11. 112 A. 144 P. 24 sq. yd. by 5.

12. A man walked 376 mi. 276 rd. in 12 days; what was the average distance each day?

13. A miner divided 37 lb. 10 oz. 17 pwt. 20 gr. of gold among his 9 sisters; how much did each receive?

14. Divide 78 bu. 3 pk. by 8 bu. 3 pk.

15. Divide 76 gal. 2 qt. by 12 gal. 3 qt.

16. Divide 43 lb. 12 oz. by 8 lb. 12 oz.

DIFFERENCE OF TIME.

ORAL EXERCISES.

1. How many hours from 8 o'clock Monday morning to 10 o'clock Monday evening?

2. How many hours from 4 o'clock Sunday afternoon to 10 o'clock Monday evening?

3. How many hours from half after 3 o'clock Saturday afternoon to half after 8 o'clock Sunday evening?

4. What time between half after 7 o'clock Tuesday morning to quarter before 10 o'clock Wednesday evening?

5. What time is it from 6.25 A. M. Friday to 6.15 P. M. Sunday?

6. A boy starts for school at half after 8 o'clock, and returns half after 4 o'clock; how long is he away from home?

7. A man begins work on the morning of the 10th, and ends it on the evening of the 15th; how much does he earn at the rate of $3 a day?

8. How many days from May 1st to June 1st, inclusive? From April 10th to May 26th, exclusive?

WRITTEN EXERCISES.

9. Find the time from April 14th to July 20th, including only one of these dates.

SOLUTION.—After April 14th there still remain in April 16 days; in May there are 31, and in June 30 days; adding these three numbers, and adding in the 20 days in July, we have 97 days.

OPERATION.

Apr. 14 to Apr. 30, 16 days.
May, 31 "
June, 30 "
July, 20 "
Ans. 97 days.

How many days from

10. Mar. 20th to June 16?
11. Apr. 25th to Aug. 12?
12. July 18th to Nov. 12?
13. Feb. 16th to July 9th?
14. Aug. 25th to Dec. 10th?
15. Oct. 11th to Apr. 9th?
16. Sept. 12th to May 9th?
17. Dec. 15th to Oct. 6th?

TIME BETWEEN DATES.

1. How many years, months, and days from Oct. 16th, 1890, to May 20th, 1895?

SOLUTION.—We write the number of the year, month, and day of both periods, and subtract the one from the other, as shown in the margin, reckoning 30 days to a month and 12 months to a year.

OPERATION.

	yr.	mo.	da.
	1895	5	20
	1890	10	16
	4	7	4

WRITTEN EXERCISES.

2. How long from Feb. 5th, 1892, to Nov. 10th, 1893?

3. How long from April 30th, 1889, to Jan. 15th, 1895?

4. How long from July 15th, 1893, to May 12th, 1896?

5. How long from Nov. 12th, 1892, to Mar. 29th, 1894?

6. How long has a note to run which is dated Dec. 30th, 1894, and made payable Jan. 16th, 1896?

7. How long has a note to run which is dated Jan. 16th, 1892, and made payable July 10th, 1893?

8. How long has a note to run which is dated Sept. 19th, 1894, and made payable Dec. 16th, 1895?

9. The Revolution began the 19th of April, 1775, and terminated January 20th, 1783; how long did it continue?

AGES OF THE PRESIDENTS.

WRITTEN EXERCISES.

1. Washington was born Feb. 22d, 1732, and died Dec. 14, 1799; what was his age?

2. John Adams was born the 30th of October, 1735, and died the 4th of July, 1826; required his age.

3. Thomas Jefferson was born April 2d, 1743, and died July 4th, 1826; what was his age?

4. James Madison was born March 16th, 1751, and died June 28th, 1836; required his age.

5. James Monroe was born April 28th, 1758, and died July 4th, 1831; required his age.

6. John Quincy Adams was born July 11th, 1767, and died Feb. 23d, 1848; what was his age?

7. Andrew Jackson was born March 15th, 1767, and died June 8th, 1845; required his age.

8. Martin Van Buren was born Dec. 5th, 1782, and died July 24th, 1862; required his age.

9. William Henry Harrison was born Feb. 9th, 1773, and died April 4th, 1841; required his age.

10. John Tyler was born Mar. 29th, 1790, and died Jan. 18th, 1862; required his age.

11. James K. Polk was born Nov. 2d, 1795, and died June 15th, 1849; required his age.

12. General Zachary Taylor was born Nov. 24th, 1784, and died July 9th, 1850; required his age.

13. Millard Fillmore was born Feb. 7th, 1800, and died March 7th, 1874; required his age.

14. Franklin Pierce was born Nov. 23d, 1804, and died Oct. 8th, 1869; required his age.

15. James Buchanan was born Apr. 23, 1791, and died June 1st, 1868; required his age.

16. Abraham Lincoln was born Feb. 12th, 1809, and died April 15th, 1865; required his age.

17. Andrew Johnson was born Dec. 29th, 1808, and died July 31st, 1875; required his age.

18. General Ulysses S. Grant was born Apr. 27th, 1822, and died July 23d, 1885; required his age.

19. Rutherford B. Hayes was born Oct. 4th, 1822, and died Jan. 17th, 1893; required his age.

20. James A. Garfield was born Nov. 19th, 1831, and died Sept. 19th, 1881; required his age.

21. Chester A. Arthur was born Oct. 5th, 1830, and died Nov. 18th, 1886; required his age.

MISCELLANEOUS EXAMPLES.

1. If the pulse beats 75 times a minute, how often does it beat in a day?

2. How long will it take to count a million, at the rate of a hundred a minute, working 12 hours a day?

3. If the distance around the earth is 25000 miles, how long will it take to walk the distance, walking 4 miles an hour?

4. How many times will a clock that ticks seconds, tick in one day?

5. A little girl picked $2\frac{1}{2}$ pecks of berries and sold them at 5 cents a pint; what did she receive?

6. How many crayons are there in 25 boxes, if each box contains one gross?

7. How many vials, holding 2 gills each, can be filled from a gallon of brandy?

8. If you are 10 years old, how many minutes have you lived, allowing $365\frac{1}{4}$ days to a year?

9. How many doses of medicine, at 6 gr. each, can be made from 4 drams?

10. If 12 of Henry's peaches fill a quart measure, how many will there be in a bushel?

11. How many square rods in a rectangular field 32 rods long and 12 rods wide?

12. How many square feet in a board 18 feet long and $2\frac{1}{2}$ feet wide?

13. How many cubic feet in a block of stone 6 feet long, 3 feet wide, and 2 feet thick?

14. Required the value of a rectangular lot 36 rods long, and 20 rods wide, at \$3 a square rod.

15. How many cords in a pile of wood 48 feet long, 4 feet wide, and 4 feet high?

16. How many cords in a pile of wood 16 feet long, 8 feet high, and 4 feet wide?

17. What must I pay for a pile of wood 24 feet long, 12 feet high, and 4 feet wide, at $1.50 a cord?

18. How much time is wasted by taking an hour's nap each afternoon for 24 years of 365 days each?

19. When apples sell at 16 cents a half-peck, what are they worth a bushel?

20. What will it cost to pave 75 sq. yards of walk at 50 cents a square foot?

21. At 5 cents a half-pint, how much does a milkman receive for 25 gallons of cream?

22. A grocer sold 8 bushels of chestnuts at 6 cents a quart; what did he receive for them?

23. What will 12 pounds of drugs cost, at the rate of $5.12 an ounce?

24. A grocer bought 132 eggs at 18 cents a dozen; what did they cost?

25. How many steps, of 3 feet each, will a person take in walking $2\frac{1}{2}$ miles?

26. A grocer bought 16 barrels of beef, 200 lb. each, at $10\frac{1}{2}$ cents a pound; what did it cost him?

27. How much will 2 A. 20 P. of land cost, at $\$2\frac{1}{2}$ a perch?

28. A man bought 32 reams of paper, at $18\frac{3}{4}$ cents a quire; what was the cost?

29. How many acres are there in a lot of land 160 rods long and 80 rods wide?

30. What cost 2 barrels of alcohol, each containing $31\frac{1}{2}$ gallons, at $3\frac{1}{2}$ cents a gill?

31. How much will I get for 16 gross of pins at $1\frac{1}{2}$ cents for each pin?

32. If a druggist uses 2 ℥ 5 ʒ of drugs daily, how much will he use in a week?

33. If blackberries are worth $3.20 a bushel, what are they worth a quart?

34. How many half-pint bottles will 2 gallons of ink fill?

35. What will 20 gross of lead-pencils cost at $62\frac{1}{2}$ cents a dozen?

36. How many quart-baskets will 2 bu. 2 qt. of strawberries fill?

37. How many ounces of calomel will it take to make 384 pills of 5 grains each?

38. Dr. Hess made calomel pills of 5 grains each, in all 2 ℥ 2 ʒ 1 ℈; how many pills did he make?

39. At $1.50 a bushel, what will a farmer receive for 2400 pounds (60 lb. = 1 bu.) of clover-seed?

40. How many panels of fence, each 10 ft., will enclose a field which is 40 rods long and 30 rods wide?

41. What will 2 bu. 3 pk. 6 qt. of shellbarks cost, at 12 cents a quart?

42. How many sheets of paper in $2\frac{1}{2}$ reams and $3\frac{1}{2}$ quires?

43. A farmer put up $1\frac{3}{8}$ miles of fence, at $\$1\frac{1}{2}$ a rod; what did it cost?

44. How many bushels in a load of corn which, at 75 cents a bushel, cost $32.25?

45. How many bushels of grapes, at 15 cents a quart, can be bought for $8.80?

46. How many miles of fence, at $1.50 a rod, can be put up for $1200?

47. A man sold 2520 lb. (60 lb. = 1 bu.) of wheat at $\$1.87\frac{1}{2}$ per bushel; what did he get for the whole?

48. What will 36 packages of paper cost, at $16\frac{2}{3}$ cents a quire, if each package contains 2 reams?

SECTION X.

PERCENTAGE.

168. The term *per cent.* means *by the hundred.* Thus, 5 per cent. of anything means 5 of a hundred of it.

169. The words *per cent.* are expressed by the symbol %. Thus, 5% means 5 per cent.; 6% means 6 per cent.

170. Since per cent. means so many of the hundred, it is evident that—

1% of a number $= \frac{1}{100}$ of it; 4% of a number $= \frac{4}{100}$ or $\frac{1}{25}$ of it;
2% " " $= \frac{2}{100}$ of it; 5% " " $= \frac{5}{100}$ or $\frac{1}{20}$ of it.

ORAL EXERCISES.

1. What is 25% of \$40?

SOLUTION.—25% of \$40 equals $\frac{25}{100}$ or $\frac{1}{4}$ of \$40, which is \$10.

2. What is 20% of 60? 25% of 48? 50% of 64? 10% of 80? 40% of 60? 20% of 75? 16% of 64?

3. What is 15% of 25? 20% of 80? 24% of 75? 40% of 100? 75% of 24? $12\frac{1}{2}$% of 64? $33\frac{1}{3}$% of 54?

4. I bought a horse for \$150, and sold it at a gain of 20%; what was my gain?

5. I sold my bicycle, which cost \$120, at a loss of 25%; what did I receive for it?

6. From a school of 160 pupils 40% were absent one rainy day; how many pupils were present?

7. 12 is 20% of what number?

SOLUTION.—If 12 is 20% or $\frac{1}{5}$ of some number, $\frac{5}{5}$ of the number equals 5 times 12, or 60.

8. 18 is 20% of what number? 16 is 25% of what number? 30 is 10% of what number? 24 is 50% of what number? 12 is 40% of what number?

9. In a certain school 15 pupils were absent, which was 10% of the number registered; how many pupils were registered?

10. 12 is what per cent. of 60?

SOLUTION.—60 is 100 per cent. of 60, and 12, which is $\frac{1}{5}$ of 60, is $\frac{1}{5}$ of 100%, or 20% of 60.

11. What per cent. of 30 is 15? of 16 is 4? of 50 is 10? of \$72 are \$9? of 48 yd. are 16 yd.

12. James having 60 marbles gave his brother 15 marbles; what per cent. did he give away?

WRITTEN EXERCISES.

1. What is 5 per cent. of \$250?

SOLUTION.—5% of \$250 is $\frac{5}{100}$ of \$250, or .05 times \$250, which, by multiplying, we find is \$12.50.

OPERATION.

\$250
.05
\$12.50

What is

2. 5% of \$280?	**5.** 9% of 364 lb.?	**8.** $12\frac{1}{2}$% of 320 oz.?
3. 6% of \$190?	**6.** 35% of 1236 h.?	**9.** $16\frac{2}{3}$% of 630 yd.?
4. 8% of 125 yd.?	**7.** 40% of 2467 lb.?	**10.** 75% of 480 bu.?

11. A man bought goods for \$580 and sold them at a gain of 8%; what was his gain?

12. A lady bought 360 acres of land and sold $12\frac{1}{2}$ of it; how much did she retain?

13. A man bought a house for \$4800 and sold it at a gain of 15%; what did he receive for it?

14. If there were 640 gallons of oil in a tank, and 24% of it leaked out, how much oil was left?

15. A man spends \$500 a year, which is 25% of his salary; what is his salary?

16. A school has 150 scholars enrolled, but the attendance is only 120; what is the percentage of attendance?

17. A man has \$960 in bank after drawing out 24% of his deposit; how much did he draw out?

18. A boy spent \$25 and then had \$125; what per cent. of his money did he spend?

19. My salary is \$1500 a year; I spend 25% of it for clothing and 35% for living; how much do I save?

SIMPLE INTEREST.

171. **Interest** is money charged for the use of money.

Thus, if Mr. Smith borrowed \$800 of Mr. Jones, Mr. Jones would charge Mr. Smith a certain number of dollars on \$100—say \$6—a year for the use of the \$800.

172. The number of dollars charged for the use of \$100 is called the **Rate of Interest.**

Thus, if \$6 is charged for the use of \$100, the rate is said to be 6 on a hundred, or 6 *per cent.*

173. **Per Cent.** means *by the hundred.* The symbol for per cent is %.

Thus, 6% means 6 per cent.; 5% means 5 per cent., etc.

METHOD BY YEARS.

1. What is the interest on \$800 for 3 yr. 6 mo. at 6%?

SOLUTION.—To find the interest for 1 year at 6%, we multiply \$800 by .06, which gives \$48. If the interest for 1 yr. is \$48 for 3 yr. 6 mo., or $3\frac{1}{2}$ yr., the interest is $3\frac{1}{2}$ times \$48, or \$168. From this we have the following method:

OPERATION.

\$800
.06
\$48.00
$3\frac{1}{2}$
\$168.00

Multiply the principal by the rate per cent., expressed decimally, and that product by the time expressed in years.

This method is practical when the months and days are a simple fractional part of a year.

WRITTEN EXERCISES.

Required the interest of

2. \$750 for 3 yr. at 4%.
3. \$834 for 4 yr. at 5%.
4. \$27.84 for 5 yr. at 6%.
5. \$36.25 for 4 yr. at 7%.
6. \$108.36 for 7 yr. at 4%.
7. \$150.44 for 6 yr. at 5%.
8. \$180 for 3 yr. 6 mo. at 7%.
9. \$470 for 7 yr. 8 mo. at 6%.
10. \$172 for 5 yr. 9 mo. at 5%.
11. \$480 for 5 yr. 4 mo. at 4%.

SIX PER CENT. METHOD.

174. The **Six Per Cent. Method** is so called because the method is based on that rate.

1. What is the interest for $240 for 2 yr. 8 mo. 12 da. at 6%?

SOLUTION.—2 yr. 8 mo. equal 32 mo. The interest of $1 for 12 mo. is 6 cts., and for 1 mo. it is $\frac{1}{12}$ of 6 cts., or $\frac{1}{2}$ ct., and for 32 mo. it is $32 \times \frac{1}{2}$ ct. = 16 cts. Since the interest on $1 for 1 mo., or 30 da., is $\frac{1}{2}$ ct., or 5 mills, for 1 da. it is $\frac{1}{30}$ of 5 mills, or $\frac{1}{6}$ of a mill, and for 12 da. it is $12 \times \frac{1}{6}$ mills = 2 mills. Hence the interest on $1 for 32 mo. and 12 da. is 16 cts. plus 2 mills, or $0.162. If the interest on $1 is $0.162, on $240 it is 240 times $0.162, which equals $38.88. From this we have the following method:

OPERATION.

2 yr. 8 mo = 32 mo.

$32 \times \frac{1}{2} = \$0.16$

$12 \times \frac{1}{6} = .002$

$\$0.162$

240

$\$38.88$

1. *Take* ONE-HALF *of the number of months as cents, and* ONE-SIXTH *of the number of days as mills; their sum will be the interest of* $1 *for the given time at* 6%.

2. *Multiply this by the principal, and the product will be the interest at* 6 *per cent. For any other rate take as many sixths of it as that rate is of six.*

When the time is brief, the rule of business men is as follows: "*Multiply dollars by days, and divide by* 6000."

WRITTEN EXERCISES.

Required the interest of

2. $480 for 8 mo. 18 da. at 6%.

3. $256 for 4 mo. 24 da. at 6%.

4. $27.60 for 1 yr. 6 mo. 12 da. at 5%.

5. $841.35 for 2 yr. 5 mo. 24 da. at 4%.

6. $48.25 for 3 yr. 6 mo. 6 da. at 4%.

7. $50.50 for 4 yr. 10 mo. 18 da. at 7%.

8. $56.50 for 5 yr. 7 mo. 24 da. at 5%.

9. What is the interest of $360 for 2 yr. and 6 mo. at 6 per cent.?

ANSWERS TO WRITTEN EXERCISES.

Page 97.—1. XXIV. **2.** XXXVII. **3.** XLVI. **4.** XXXVIII. **5.** LXIX. **6.** XLVIII. **7.** LXXXVII. **8.** LXXVIII. **9.** XCVI. **10.** CCVI. **11.** CCCLVI. **12.** DLXXVIII. **13.** MLII. **14.** MCDLXX. **15.** VIII.

Page 98.—2. 77. **3.** 78. **4.** 98. **5.** 866. **6.** 679. **7.** 979. **8.** 987. **9.** 889. **10.** 789. **11.** 979. **12.** 989. **13.** 6987. **14.** 6968. **15.** 9899. **16.** 6487. **17.** 6677. **18.** 6578. **19.** 37364. **20.** 88785. **21.** 35765. **22.** 99998.

Page 99.—1. $875. **2.** 989 steps. **3.** $295. **4.** 788 papers. **5.** 738 pupils.

Page 100.—6. 767 stars. **7.** 788 cattle. **8.** $488. **9.** 5989 bushels. **10.** $8698. **Art. 25.—2.** 144. **3.** 125. **4.** 156. **5.** 115. **6.** 177. **7.** 179. **8.** 967. **9.** 1596. **10.** 1377. **11.** 1939. **12.** 1498. **13.** 1848.

Page 101.—14. 2104. **15.** 1944. **16.** 1825. **17.** 1708. **18.** 1989. **19.** 2087. **20.** 1683. **21.** 2819. **22.** 1267. **23.** 913. **24.** 16760. **25.** 15930. **26.** 20498. **27.** 15718. **28.** 7309. **29.** 6808. **30.** 14664. **31.** 15351.

Page 102.—2. 162 miles. **3.** 49 examples. **4.** 1235 steps. **5.** 168 pounds. **6.** 76 turkeys.

Page 103.—7. 106 miles. **8.** $8.25. **9.** 196. **10.** $5.42. **11.** $2.74. **12.** 113 flowers. **13.** 225 words. **14.** $2.12. **15.** 7811 cows. **16.** 209 words. **17.** $2.62. **18.** 1930 bushels.

Page 104.—2. 542. **3.** 6321. **4.** 1221. **5.** 220. **6.** 4213. **7.** 2321. **8.** 711. **9.** 312. **10.** 203. **11.** 702. **12.** 520. **13.** 531. **14.** 5512. **15.** 2734. **16.** 5024. **17.** 6257. **18.** 1361. **19.** 4623.

Page 105.—20. 4066. **21.** 26016. **22.** 6415. **23.** 35422. **24.** 77443. **25.** 161006. **26.** 741551. **27.** 21353. **28.** 44516. **29.** 57234.

Page 106.—**2.** 32. **3.** 2342. **4.** 3014. **5.** 5750. **6.** 4513. **7.** 7721. **8.** $2.23. **9.** 1323 bushels.

Page 107.—**2.** 38. **3.** 15. **4.** 28. **5.** 27. **6.** 37. **7.** 37 **8.** 56. **9.** 48. **10.** 28. **11.** 54. **12.** 29. **13.** 48. **14.** 48. **15.** 67. **16.** 9. **17.** 62. **18.** 27. **19.** 27. **20.** 124. **21.** 335. **22.** 718. **23.** 736. **24.** 104. **25.** 308. **26.** 317. **27.** 352. **28.** 235. **29.** 351. **30.** 456. **31.** 355. **32.** 144. **33.** 373. **34.** 592. **35.** 688. **36.** 463. **37.** 273.

Page 108.—**38.** 185. **39.** 466. **40.** 447. **41.** 415. **42.** 331. **43.** 274. **44.** 468. **45.** 122. **46.** 448. **47.** 165. **48.** 167. **49.** 466. **50.** 3018. **51.** 5638. **52.** 3344. **53.** 6436. **54.** 442. **55.** 4635. **56.** 13231. **57.** 42493. **58.** 35433. **59.** 31628. **60.** 9058. **61.** 20071. **62.** 32715. **63.** 49213. **64.** 34655. **65.** 15621. **66.** 35375. **67.** 99999.

Page 109.—**2.** 208 chickens. **3.** 357 feet. **4.** 144 steps. **5.** 68. **6.** 105 chestnuts. **7.** $5485. **8.** 27 pins. **9.** 18 robins. **10.** $2.25.

Page 110.—**11.** 39 roses. **12.** 178 words. **13.** 808 steps. **14.** 337 cents. **15.** 618 feet. **16.** 186 words. **17.** 25 cents. **18.** $2.25. **19.** $175. **20.** 37 quarts. **21.** 1435 bushels.

Page 111.—**1.** $1.23. **2.** 341 words. **3.** 169 chickens. **4.** $71. **5.** 207 acres. **6.** $183. **7.** 35 cents. **8.** 62 cents. **9.** 155. **10.** 108 cents. **11.** 219 chestnuts. **12.** 242 steps. **13.** 611 feet.

Page 112.—**1.** $1.22. **2.** $1.35. **3.** $1.66. **4.** $0.44. **5.** $1.25. **6.** $87. **7.** $10. **8.** $9310. **9.** $14. **10.** $8. **11.** $82. **12.** $1.35.

Page 113.—Ans. vary, except the following: **7.** 34 years. **9.** 117 years. **14.** 67 years.

Page 114.—**2.** 162. **3.** 152. **4.** 172. **5.** 280. **6.** 258. **7.** 504.

Page 115.—**8.** 170. **9.** 228. **10.** 470. **11.** 266. **12.** 540. **13.** 480. **14.** 550. **15.** 648. **16.** 688. **17.** 960. **18.** 888. **19.** 1080. **20.** 702. **21.** 1744. **22.** 2865. **23.** 4092. **24.** 6328. **25.** 6440. **26.** 4368. **27.** 5845. **28.** 8172. **29.** 5888. **30.** 4459. **31.** 3990. **32.** 7760. **33.** 6372. **34.** 4260. **35.** 7194. **36.** 2496. **37.** 5472. **38.** 10875. **39.** 13848. **40.** 28686. **41.** 17541. **42.** 48601. **43.** 61984. **44.** 537846. **45.** 377480. **46.** 566736. **47.** 481131. **48.** 355284. **49.** 594730. **50.** 786888. **51.** 405276. **52.** 713896. **53.** 924360. **54.** 774466. **55.** 1004508.

Page 116.—**2.** 130 words. **3.** 210 miles.

Page 117.—**4.** 120 marbles. **5.** 750. **6.** $1.75. **7.** 448 roses. **8.** $7.83. **9.** 396 lines. **10.** 105 marbles. **11.** 300 minutes. **12.** 288 pupils. **13.** $192. **14.** 63360 feet. **15.** 4368. **16.** 6075. **17.** 28116. **18.** 675252. **19.** $20. **20.** 310 hours.

Page 118.—**2.** 805. **3.** 1104. **4.** 1792. **5.** 2210. **6.** 3375. **7.** 2220. **8.** 3192. **9.** 1634. **10.** 4800. **11.** 1672. **12.** 6460. **13.** 5226. **14.** 7935. **15.** 19909. **16.** 19145. **17.** 32568. **18.** 52800. **19.** 47628. **20.** 32625. **21.** 53105. **22.** 53650. **23.** 69574. **24.** 48776. **25.** 48720.

Page 119.—**26.** 82460. **27.** 181288. **28.** 111078. **29.** 221004. **30.** 261553. **31.** 267840. **32.** 42435. **33.** 128340. **34.** 908544. **35.** 1540350. **36.** 728178. **37.** 1573088. **38.** 486461. **39.** 3489792. **40.** 1979325. **41.** 5293932. **42.** 3142304. **43.** 4723478. **44.** 879120. **45.** 1609930, **46.** 4560204. **47.** 1289343. **48.** 3670191. **49.** 4021248. **50.** 9039345. **51.** 9217902. **52.** 24374500. **53.** 10147958. **54.** 10355445. **55.** 9875732.

Page 120.—**2.** 1050 miles. **3.** $6000. **4.** 1012 trees. **5.** 3825 bushels. **6.** 288 oranges. **7.** 6552 pages. **8.** 11736 sheep. **9.** 108432.

Page 121.—**10.** $24885. **11.** 18000 times. **12.** $52.50. **13.** 525600 minutes. **14.** $18750. **15.** 1411200 grains. **Art. 39.**—**2.** 111935. **3.** 212784. **4.** 287854. **5.** 182972. **6.** 433832. **7.** 12992300. **8.** 4623530. **9.** 23082572. **10.** 23350278. **11.** 25260730. **12.** 32448768. **13.** 35469588.

Page 122.—**15.** 1080000. **16.** 2340000. **17.** 2736000. **18.** 35700000. **19.** 47523000. **20.** 16450000. **21.** 6273000. **22.** 21350000. **23.** 56640000. **24.** 1551420000.

Page 123.—**2.** $36. **3.** $44.20. **4.** $24.05. **5.** $2664. **6.** 12936 cubic inches. **7.** 823680 feet. **8.** 1766600 square yards.

Page 124.—**2.** 164. **3.** 286, +1. **4.** 378. **5.** 150. **6.** 174, +2. **7.** 226. **8.** 178, +1. **9.** 265. **10.** 156. **11.** 213. **12.** 176. **13.** 133, +3. **14.** 130, +1. **15.** 169, +3. **16.** 156. **17.** 175. **18.** 147. **19.** 188, +4. **20.** 143. **21.** 130, +5. **22.** 109, +5. **23.** 142, +1. **24.** 131. **25.** 1142, +2. **26.** 122. **27.** 136. **28.** 142, +4. **29.** 112. **30.** 121. **31.** 1401. **32.** 54, +1. **33.** 55, +7. **34.** 92. **35.** 571. **36.** 709. **37.** 808, +1. **38.** 78. **39.** 65. **40.** 94, +4. **41.** 408. **42.** 513, +7. **43.** 807, +1.

Page 125.—**2.** 34 pigs. **3.** 59 hours. **4.** 87 yards. **5.** 86 barrels.

Page 126.—**6.** 69 minutes. **7.** 49 weeks. **8.** 81 hours. **9.** 165 jackets. **10.** $600. **11.** 537 persons. **12.** 107 tons. **13.** 1209 fathoms. **14.** 84 weeks. **15.** 48 dozen and 4. **16.** 31 times and 2 quarts over. **17.** 440 times. **18.** 80 weeks. **19.** 144 feet. **20.** 7 miles.

Page 127.—**2.** 34. **3.** 62. **4.** 28. **5.** 35. **6.** 53. **7.** 56. **8.** 46, +10. **9.** 29, +12. **10.** 36. **11.** 38. **12.** 180. **13.** 221. **14.** 114. **15.** 240. **16.** 197. **17.** 210. **18.** 302. **19.** 228, +21. **20.** 180, +1. **21.** 154, +16. **22.** 206, +18. **23.** 120, +24. **24.** 85, +33. **25.** 164, +40. **26.** 142, +8. **27.** 161, +52.

Page 128.—**2.** 70 cows. **3.** 13 pounds. **4.** 15 bushels. **5.** 22 hours. **6.** 300 days. **7.** 85. **8.** 76 pounds.

Page 129.—**9.** 69 pounds. **10.** 25 trees. **11.** 4 cents. **12.** 12 miles. **13.** 4 cents. **14.** $21. **15.** 32 cents. **16.** 250 days. **Art. 50.**—**1.** 53. **2.** 66. **3.** 35. **4.** 129, +3. **5.** 72. **6.** 88. **7.** 46. **8.** 113, +28. **9.** 45. **10.** 133. **11.** 68. **12.** 119, +77. **13.** 115. **14.** 85. **15.** 313. **16.** 414. **17.** 279, +24. **18.** 651. **19.** 144. **20.** 503. **21.** 218. **22.** 375. **23.** 502, +4. **24.** 558, +108.

Page 130.—**25.** 96. **26.** 27 acres. **27.** $50. **28.** $35. **29.** 40 days. **30.** 200 days. **Art. 51.**—**2.** 3, +24. **3.** 4, +57. **4.** 6, +56. **5.** 12. **6.** 3, +25. **7.** 12, +442. **8.** 12, +490. **9.** 15, +476. **10.** 28, +144. **11.** 39, +34. **12.** 17, +132. **13.** 226, +120. **14.** 7, +1254. **15.** 17, +37. **16.** 25, +2220. **17.** 16, +6840.

Page 131.—**1.** 225 pounds. **2.** 246 bushels. **3.** 79 cows. **4.** 110 horses. **5.** 95 days. **6.** 237 bushels. **7.** 63 gallons. **8.** 48 carriages. **9.** 32 seconds. **10.** 5000 minutes. **1.** 2502. **2.** 3056. **3.** 300 pigs. **4.** $444.

Page 132.—**5.** 508 acres. **6.** $382. **7.** $9272. **8.** $576. **9.** 2016 miles. **10.** $636. **11.** $2200. **12.** 200 teachers. **13.** $12040. **14.** $1675. **15.** $3616.

Page 133.—**1.** 115 years. **2.** 128 years. **4.** 155 years. **6.** 168 years. **10.** 26 years. **12.** 38 years. **14.** 32 years. **15.** 85 years.

Page 134.—**2.** 1148 years. **4.** 504 years. **5.** 956 years. **6.** 165 years. **7.** 248 years. **8.** 657 years. **9.** 522 years. **10.** 15 years. **11.** 335 years. **12.** 16 years. **13.** 728 years. **14.** 1027 years. **15.** 46 years.

Page 137.—2. 1155. **3.** 17955. **4.** 1225; 3249; 11881; 26569.

Page 138.—5. 343; 1331; 4913; 15625. **6.** 625; 28561; 50625; 331776. **7.** 2310. **8.** 17280. **9.** 6, 10, 15. **10.** 15, 21, 33, 35, 55, 77, 105, 165, 231, 385, 1155.

Page 139.—2. 2, 2, 2, 2, 3. **3.** 2, 2, 2, 3, 3. **4.** 3, 3, 3, 3. **5.** 2, 2, 3, 3, 3. **6.** 5, 5, 7. **7.** 2, 3, 3, 3, 5. **8.** 3, 3, 5, 7. **9.** 2, 2, 2, 2, 3, 7. **10.** 5, 5, 19. **11.** 2, 3, 11, 13. **12.** 3, 3, 5, 5, 7. **13.** 2, 2, 3, 3, 3, 7, 11. **14.** 2, 2, 2, 2, 3, 5, 5. **15.** 2, 3, 3, 3, 3, 3, 3, 5. **16.** 3, 3, 3, 5, 7, 31. **17.** 2, 3, 5, 5, 7, 7, 11.

Page 141.—2. 6. **3.** 30. **4.** 22. **5.** 12. **6.** 35. **7.** 3. **8.** 6. **9.** 36. **10.** 42. **11.** 42. **12.** 6 feet. **13.** 16 yards. **14.** 24 feet. **15.** 25 bushels. **16.** $30.

Page 143.—2. 60. **3.** 90. **4.** 144. **5.** 144. **6.** 135. **7.** 120. **8.** 180. **9.** 360. **10.** 6048. **11.** 945. **12.** 120 cents. **13.** 60 yards.

Page 144.—2. $1\frac{3}{5}$. **3.** $1\frac{1}{5}$. **4.** $1\frac{1}{7}$. **5.** $1\frac{2}{5}$. **6.** $1\frac{4}{5}$. **7.** $1\frac{1}{5}$. **8.** 2. **9.** 14. **10.** $1\frac{23}{35}$. **11.** 8. **12.** 6. **13.** 3.

Page 145.—14. 18. **15.** 30. **16.** $1\frac{13}{15}$. **17.** 54 cows. **18.** 12 pigs. **19.** 10 tons. **20.** 60 bushels. **21.** 16 yards. **22.** 16 yards. **23.** $58\frac{1}{3}$ cents. **24.** 8 pieces. **25.** $\$2.91\frac{2}{3}$.

Page 151.—2. $\frac{23}{5}$. **3.** $\frac{31}{4}$. **4.** $\frac{59}{6}$. **4.** $\frac{61}{8}$. **6.** $\frac{71}{8}$. **7.** $\frac{77}{8}$. **8.** $\frac{111}{8}$. **9.** $\frac{131}{9}$. **10.** $\frac{111}{7}$. **11.** $\frac{71}{4}$. **12.** $\frac{93}{5}$. **13.** $\frac{65}{3}$. **14.** $\frac{239}{12}$. **15.** $\frac{3347}{13}$. **16.** $\frac{5775}{16}$. **17.** $\frac{4911}{18}$. **18.** $\frac{10389}{20}$. **19.** $\frac{17721}{25}$. **20.** $\frac{26981}{37}$. **21.** $\frac{88769}{99}$.

Page 152.—2. $2\frac{1}{4}$. **3.** $3\frac{2}{3}$. **4.** $3\frac{4}{5}$. **5.** 8. **6.** 12. **7.** 9. **8.** $11\frac{2}{3}$. **9.** $10\frac{5}{7}$. **10.** $7\frac{5}{6}$. **11.** $8\frac{4}{11}$. **12.** $8\frac{4}{8}$. **13.** $7\frac{1}{11}$. **14.** $8\frac{6}{12}$. **15.** $13\frac{14}{17}$. **16.** $34\frac{10}{21}$. **17.** $14\frac{5}{15}$. **18.** 12. **19.** $13\frac{12}{24}$. **20.** 11. **21.** 11.

Page 154.—2. $\frac{8}{12}$. **3.** $\frac{25}{30}$. **4.** $\frac{14}{18}$. **5.** $\frac{21}{27}$. **6.** $\frac{18}{20}$. **7.** $\frac{27}{30}$. **8.** $\frac{8}{30}$. **9.** $\frac{20}{25}$. **10.** $\frac{32}{36}$. **11.** $\frac{63}{81}$. **12.** $\frac{33}{36}$. **13.** $\frac{56}{60}$.

Page 155.—2. $\frac{2}{3}$. **3.** $\frac{2}{3}$. **4.** $\frac{5}{6}$. **5.** $\frac{2}{3}$. **6.** $\frac{4}{7}$. **7.** $\frac{2}{3}$. **8.** $\frac{3}{4}$. **9.** $\frac{5}{8}$. **10.** $\frac{2}{3}$. **11.** $\frac{4}{5}$. **12.** $\frac{3}{4}$. **13.** $\frac{7}{8}$. **14.** $\frac{9}{10}$. **15.** $\frac{11}{12}$. **16.** $\frac{12}{13}$. **17.** $\frac{3}{5}$. **18.** $\frac{7}{8}$. **19.** $\frac{9}{10}$. **20.** $\frac{11}{12}$. **21.** $\frac{12}{13}$.

Page 156.—2. $\frac{10}{15}$, $\frac{12}{15}$. **3.** $\frac{24}{30}$, $\frac{25}{30}$. **4.** $\frac{8}{10}$, $\frac{9}{10}$. **5.** $\frac{21}{24}$, $\frac{22}{24}$. **6.** $\frac{27}{36}$, $\frac{22}{36}$. **7.** $\frac{80}{90}$, $\frac{81}{90}$. **8.** $\frac{20}{48}$, $\frac{33}{48}$. **9.** $\frac{55}{60}$, $\frac{52}{60}$. **10.** $\frac{33}{36}$, $\frac{34}{36}$. **11.** $\frac{52}{80}$, $\frac{51}{80}$. **12.** $\frac{51}{60}$, $\frac{42}{60}$. **13.** $\frac{168}{182}$, $\frac{169}{182}$. **14.** $\frac{8}{12}$, $\frac{9}{12}$, $\frac{10}{12}$. **15.** $\frac{18}{24}$, $\frac{20}{24}$, $\frac{21}{24}$. **16.** $\frac{24}{30}$, $\frac{25}{30}$, $\frac{27}{30}$. **17.** $\frac{20}{24}$, $\frac{21}{24}$, $\frac{22}{24}$. **18.** $\frac{45}{48}$, $\frac{38}{48}$. **19.** $\frac{34}{36}$, $\frac{25}{36}$. **20.** $\frac{46}{40}$, $\frac{27}{40}$. **21.** $\frac{38}{48}$, $\frac{28}{48}$. **22.** $\frac{46}{60}$, $\frac{31}{60}$.

Page 158.—**2.** $1\frac{1}{15}$. **3.** $1\frac{11}{20}$. **4.** $1\frac{7}{12}$. **5.** $1\frac{23}{35}$. **6.** $1\frac{3}{56}$. **7.** $\frac{83}{90}$. **8.** $1\frac{31}{45}$. **9.** $10\frac{5}{12}$. **10.** $11\frac{7}{10}$. **11.** $16\frac{1}{2}$. **12.** $9\frac{7}{8}$. **13.** $1\frac{41}{56}$. **14.** $1\frac{1}{30}$. **15.** $1\frac{11}{21}$. **16.** $1\frac{11}{15}$. **17.** $1\frac{73}{140}$. **18.** $1\frac{17}{68}$. **19.** $1\frac{193}{220}$. **20.** $2\frac{1}{4}$. **21.** $2\frac{1}{24}$. **22.** $2\frac{9}{20}$. **23.** $1\frac{5}{12}$. **24.** $1\frac{11}{20}$. **25.** $\frac{82}{63}$. **27.** $43\frac{3}{20}$ acres. **28.** $\$154\frac{13}{30}$. **29.** $244\frac{7}{16}$ yards. **30.** $84\frac{5}{8}$ feet. **31.** $132\frac{1}{3}$ feet. **32.** $26\frac{13}{60}$ baskets.

Page 160.—**2.** $\frac{1}{12}$. **3.** $\frac{1}{12}$. **4.** $\frac{7}{30}$. **5.** $\frac{13}{35}$. **6.** $\frac{7}{24}$. **7.** $\frac{5}{84}$. **8.** $\frac{5}{24}$. **9.** $\frac{1}{56}$. **10.** $\frac{5}{126}$. **11.** $\frac{1}{24}$. **12.** $\frac{7}{30}$. **13.** $\frac{1}{60}$. **14.** $3\frac{1}{2}$. **15.** $4\frac{2}{7}$. **16.** $4\frac{1}{3}$. **17.** $6\frac{4}{11}$. **18.** $9\frac{1}{3}$. **19.** $13\frac{7}{15}$. **21.** $3\frac{2}{3}$. **22.** $2\frac{6}{7}$. **23.** $3\frac{5}{8}$. **24.** $11\frac{7}{8}$. **25.** $10\frac{7}{10}$. **26.** $12\frac{1}{24}$. **27.** $15\frac{1}{18}$. **28.** $13\frac{23}{36}$. **29.** $\$\frac{7}{20}$. **30.** $\frac{1}{20}$ quart.

Page 161.—**31.** $\frac{19}{40}$. **32.** $\frac{33}{70}$. **33.** $\frac{1}{30}$. **34.** $\frac{1}{30}$. **35.** $\frac{1}{5}$. **36.** $\frac{17}{48}$. **37.** $\frac{3}{8}$.—**1.** $\$4\frac{3}{8}$. **2.** $\$14\frac{1}{4}$. **3.** 12 pages. **4.** $\$2\frac{17}{20}$. **5.** $\$54\frac{17}{20}$. **6.** $\$5\frac{1}{10}$. **7.** $10. **8.** $\$5\frac{3}{4}$.

Page 162.—**9.** $\$30\frac{1}{20}$. **10.** $\$20\frac{3}{10}$. **11.** $\$2\frac{11}{20}$. **12.** $\$11\frac{1}{4}$.

Page 163.—**2.** $\frac{21}{32}$. **3.** $\frac{35}{54}$. **4.** $\frac{24}{35}$. **5.** $\frac{33}{27}$. **6.** $\frac{9}{16}$. **7.** $\frac{3}{4}$. **8.** $\frac{1}{2}$. **9.** $\frac{33}{56}$. **10.** $\frac{8}{9}$. **11.** $\frac{7}{9}$. **12.** $\frac{4}{9}$. **13.** $\frac{35}{64}$. **14.** $\frac{9}{20}$. **15.** $\frac{119}{240}$. **16.** $\frac{2}{9}$. **18.** $\frac{5}{9}$. **19.** $\frac{5}{8}$. **20.** $\frac{1}{2}$. **21.** $\frac{2}{3}$. **22.** $\frac{1}{2}$.

Page 165.—**2.** $4\frac{1}{2}$. **3.** $2\frac{3}{4}$. **4.** $6\frac{8}{15}$. **5.** $2\frac{16}{19}$. **6.** $8\frac{1}{2}$. **7.** $6\frac{3}{4}$. **8.** $4\frac{1}{6}$. **9.** $30\frac{1}{4}$. **10.** $67\frac{1}{2}$. **11.** $186\frac{1}{4}$. **12.** $192\frac{8}{7}$. **13.** $347\frac{7}{9}$. **14.** $3\frac{7}{9}$. **15.** $3\frac{3}{8}$.

Page 166.—**2.** $\frac{3}{4}$. **3.** $\frac{7}{9}$. **4.** $\frac{3}{4}$. **5.** $\frac{33}{56}$. **6.** $\frac{27}{32}$. **7.** $\frac{9}{10}$. **8.** $\frac{1}{3}$. **9.** $\frac{8}{9}$. **10.** $\frac{32}{45}$. **11.** $18\frac{1}{3}$. **12.** 22. **13.** $38\frac{1}{2}$. **14.** 44. **15.** 21. **16.** $19\frac{5}{6}$. **17.** $1\frac{1}{15}$. **18.** $\frac{8}{20}\frac{5}{7}$. **19.** $\frac{7}{10}$. **20.** $\frac{5}{9}$. **21.** $\frac{2}{3}$. **22.** $\frac{3}{5}$. **23.** $\frac{3}{5}$. **24.** $\frac{7}{20}$. **25.** $\frac{40}{57}$. **26.** $\frac{5}{12}$. **27.** $\frac{5}{8}$. **28.** $\frac{1}{2}$. **29.** $3\frac{7}{20}$ tons. **30.** $\$3\frac{1}{2}$. **31.** $\$13\frac{1}{8}$.

Page 167.—**32.** $\$131\frac{7}{20}$. **33.** $\$57\frac{3}{8}$. **2.** 133. **3.** 371. **4.** 504. **5.** 568. **6.** $558\frac{4}{7}$. **7.** $653\frac{3}{8}$. **8.** 770. **9.** $491\frac{3}{8}$. **10.** $306\frac{3}{5}$. **11.** $353\frac{5}{11}$. **12.** 596. **13.** 490.

Page 168.—**2.** $\frac{3}{10}$. **3.** $\frac{2}{11}$. **4.** $\frac{2}{15}$. **5.** $\frac{4}{17}$. **6.** $\frac{3}{25}$. **7.** $\frac{12}{91}$. **8.** $\frac{16}{15}$. **9.** $\frac{9}{28}$. **10.** $\frac{13}{36}$. **11.** $\frac{9}{20}$. **12.** $\frac{3}{34}$. **13.** $\frac{3}{50}$. **14.** $\frac{3}{56}$. **15.** $1\frac{1}{15}$. **16.** $\frac{5}{78}$. **17.** $\$\frac{8}{15}$. **18.** $\$\frac{1}{10}$.

Page 169.—**19.** $\$\frac{3}{10}$. **20.** $\frac{1}{35}$. **21.** $2\frac{5}{8}$ miles. **22.** $1\frac{13}{18}$ pounds.

Page 170.—**2.** $1\frac{1}{7}$. **3.** $1\frac{1}{5}$. **4.** $1\frac{9}{40}$. **5.** $2\frac{2}{5}$. **6.** $1\frac{1}{32}$. **7.** $\frac{100}{121}$. **8.** $1\frac{1}{4}$. **9.** $\frac{27}{32}$. **10.** $1\frac{33}{27}$. **11.** $2\frac{8}{17}$. **12.** $1\frac{3}{4}$. **13.** $\frac{25}{27}$. **14.** $\frac{2}{3}$. **15.** $\frac{4}{5}$. **16.** $\frac{2}{3}$. **17.** $\frac{9}{14}$. **18.** $1\frac{1}{5}$. **19.** $1\frac{2}{25}$. **20.** 6 yards. **21.** 21 times. **22.** 6 jars. **23.** 16 dolls. **24.** 26 dozen. **25.** $\frac{16}{27}$. **26.** $1\frac{1}{3}$. **27.** $1\frac{1}{3}$. **28.** $\frac{2}{5}$. **29.** $\frac{7}{9}$. **30.** $\frac{77}{77}$. **31.** $2\frac{1}{5}$.

Page 171.—3. $14\frac{1}{2}$. **4.** $13\frac{9}{10}$. **5.** $8\frac{89}{90}$. **6.** $7\frac{8}{9}$. **7.** $8\frac{31}{36}$. **8.** $11\frac{1}{3}$. **9.** $9\frac{1}{4}$. **10.** $12\frac{1}{7}$. **11.** $23\frac{1}{8}$. **12.** $13\frac{5}{9}$. **13.** 50. **14.** 8. **15.** 6. **16.** 15. **17.** $12\frac{1}{5}$. **18.** 6 cows. **19.** 7 acres. **20.** 15 bushels.

Page 172.—2. $\frac{9}{10}$. **3.** 6. **4.** 3. **5.** $1\frac{1}{3}$. **6.** $8\frac{1}{18}$. **7.** $2\frac{38}{75}$. **8.** $4\frac{389}{1000}$. **9.** $\frac{1}{4}$. **10.** $\frac{3}{20}$. **11.** $\frac{4}{5}$. **12.** $1\frac{1}{2}$. **13.** 10. **14.** $\frac{28}{35}$. **15.** $4\frac{8}{45}$. **16.** $6\frac{3}{4}$. **17.** $\frac{2}{5}$.—**Mis. Ex. 1.** $\frac{55}{8}$. **2.** $\frac{68}{7}$. **3.** $\frac{115}{9}$. **4.** $\frac{1995}{14}$. **5.** $\frac{6224}{25}$. **6.** $\frac{7447}{61}$. **7.** $\frac{31109}{48}$. **8.** $\frac{8447}{15}$. **9.** $\frac{18882}{21}$. **10.** $\frac{16190}{34}$. **11.** $\frac{6447}{18}$. **12.** $\frac{27982}{21}$. **13.** $\frac{83038}{24}$. **14.** $\frac{136996}{25}$. **15.** $\frac{831938}{107}$. **16.** $12\frac{1}{6}$. **17.** $13\frac{9}{13}$. **18.** $13\frac{5}{7}$. **19.** $3\frac{13}{48}$. **20.** $10\frac{16}{33}$. **21.** $3\frac{13}{29}$. **22.** $7\frac{223}{78}$. **23.** $6\frac{8}{23}$. **24.** $7\frac{84}{109}$. **25.** $3\frac{9}{31}$. **26.** $2\frac{157}{225}$. **27.** $5\frac{1}{113}$. **28.** $4\frac{7683}{4883}$. **29.** $14\frac{27}{52}$. **30.** $132\frac{28}{121}$.

Page 173.—31. $1\frac{1}{12}$. **32.** $1\frac{1}{12}$. **33.** $\frac{4}{5}$. **34.** $\frac{13}{14}$. **35.** $\frac{7}{9}$. **36.** $\frac{1}{4}$. **37.** $\frac{5}{7}$. **38.** $\frac{1}{8}$. **39.** $\frac{41}{62}$. **40.** $\frac{5}{7}$. **41.** $\frac{9}{14}$. **42.** $\frac{5}{18}$. **43.** $\frac{1}{2}$. **44.** $\frac{11}{14}$. **45.** $\frac{3}{4}$. **46.** $\frac{9}{14}$. **47.** $\frac{15}{28}$. **48.** $\frac{5}{8}$. **49.** $1\frac{2}{3}$. **50.** $3\frac{3}{5}$. **51.** $17\frac{1}{8}$. **52.** $\frac{2}{3}$. **53.** $\frac{7}{4}$. **54.** $1\frac{1}{5}$. **55.** $13\frac{1}{3}$. **56.** $5\frac{1}{2}$. **57.** $\frac{8}{21}$. **58.** $\frac{9}{10}$. **59.** $6\frac{99}{70}$. **60.** $61\frac{1}{2}$. **61.** $1\frac{139}{80}$. **62.** $1\frac{17}{24}$. **63.** $1\frac{13}{63}$. **64.** $10\frac{11}{20}$. **65.** $14\frac{17}{24}$. **66.** $18\frac{9}{20}$. **67.** $2\frac{1}{4}$. **68.** $2\frac{1}{24}$. **69.** $2\frac{9}{35}$. **70.** $2\frac{68}{88}$. **71.** $2\frac{23}{60}$. **72.** $2\frac{9}{16}$. **73.** $\frac{1}{18}$. **74.** $\frac{13}{24}$. **75.** $\frac{13}{42}$. **76.** $2\frac{11}{12}$. **77.** $3\frac{9}{40}$. **78.** $1\frac{17}{18}$. **79.** $1\frac{7}{24}$. **80.** $\frac{13}{72}$. **81.** $\frac{151}{168}$. **82.** $\frac{829}{860}$. **83.** $\frac{8}{1435}$. **84.** $\frac{13}{18}$. **85.** $4\frac{1}{2}$. **86.** $6\frac{3}{5}$. **87.** $10\frac{1}{2}$. **88.** 46. **89.** $94\frac{3}{8}$. **90.** $128\frac{1}{3}$. **91.** $\frac{5}{9}$. **92.** $\frac{1}{2}$. **93.** $\frac{455}{64}$. **94.** $\frac{8}{15}$. **95.** $\frac{8}{13}$. **96.** $\frac{7}{10}$.

Page 174.—97. $\frac{3}{14}$. **98.** $\frac{4}{27}$. **99.** $\frac{3}{26}$. **100.** 28. **101.** 18. **102.** $\frac{4}{9}$. **103.** $1\frac{1}{11}$. **104.** $\frac{2}{3}$. **105.** $\frac{1}{6}$. **106.** $\frac{4}{15}$. **107.** $\frac{5}{8}$. **108.** $\frac{1366}{1771}$. **109.** $\frac{1}{5}$. **110.** $\frac{1}{6}$. **111.** $\frac{2}{15}$. **112.** $\frac{21}{25}$. **113.** $1\frac{43}{72}$. **114.** $1\frac{3}{11}$. **115.** $1\frac{3}{5}$. **116.** $4\frac{428}{447}$. **117.** $\frac{7111}{10252}$. **118.** $1\frac{3}{7}$. **119.** $1\frac{3}{8}$. **120.** $1\frac{1}{2}$. **121.** $3\frac{41}{63}$. **122.** $\frac{15}{28}$. **123.** $\frac{7775}{12998}$.—**Prac. Prob. 1.** 15 cents. **2.** \$1. **3.** \$9. **4.** 84 cents. **5.** \$1.50. **6.** \$1.50. **7.** \$6.25. **8.** $90\frac{3}{8}$ cents. **9.** \$36. **10.** $\$1.49\frac{1}{2}$. **11.** \$4. **12.** $\$3.07\frac{1}{2}$.

Page 175.—13. $7\frac{1}{8}$ yards. **14.** $\$13\frac{3}{20}$. **15.** $5\frac{3}{4}$ tons. **16.** $49\frac{11}{12}$ acres. **17.** $\frac{1}{2}$. **18.** $\$17\frac{1}{24}$. **19.** $\$22\frac{3}{4}$. **20.** $27\frac{1}{3}$ bushels. **21.** $\$19\frac{1}{4}$. **22.** $33\frac{1}{3}$ bushels. **23.** 36 pounds. **24.** \$5. **25.** $10\frac{2}{3}$ tons. **26.** $5\frac{3}{5}$ cents. **27.** $3\frac{1}{3}$ yards. **28.** $9\frac{2}{5}$ yards. **29.** $151\frac{1}{2}$ bushels.

Page 176.—30. 18 sheep. **31.** $24\frac{1}{3}$ cents. **32.** $12\frac{1}{2}$ acres. **33.** 14 cents. **34.** 24 cows. **35.** $12\frac{1}{2}$ yards. **36.** $\frac{455}{1152}$. **37.** $\$6\frac{1}{4}$.—**2.** 279 cents. **3.** \$77.

Page 177.—4. \$1560. **5.** \$125. **6.** \$1008. **7.** \$21.60. **8.** $\$22\frac{2}{3}$. **9.** 1944 miles. **10.** 29220 days. **11.** 48000 acres. **12.** 10944 miles. —**2.** \$112. **3.** \$42. **4.** \$20. **5.** 90 miles.

Page 178.—6. $720. **7.** $8400. **8.** 11340 pounds. **9.** $3360. **10.** $2360.—**2.** $174. **3.** $450. **4.** $72. **5.** 50 pounds. **6.** 55 pounds. **7.** 40 pounds. **8.** 168 pounds. **9.** 165 cubic inches.

Page 181.—10. 1584 cu. in.—**2.** $150. **3.** 100 hens. **4.** $952. **5.** $960. **6.** 36 feet. **7.** $1800. **8.** 560 acres. **9.** 280. **10.** 20 miles; 28 miles.

Page 183.—2. .34. **3.** .75. **4.** .26. **5.** .025. **6.** .407. **7.** .708. **8.** 500.025. **9.** .3007.

Page 184.—10. 30.0007. **11.** .060008. **12.** 600.000008. **13.** .000496. **14.** 400.000096. **15.** 56.000078. **16.** .04076. **17.** 900.00069. **18.** 35000.000008. **19.** 90000.0000007. **20.** 18000.00000001.

Page 186.—2. $\frac{7}{20}$. **3.** $\frac{12}{25}$. **4.** $\frac{1}{8}$. **5.** $\frac{5}{8}$. **6.** $\frac{3}{4}$. **7.** $\frac{4}{5}$. **8.** $\frac{3}{8}$. **9.** $\frac{7}{8}$. **10.** $8\frac{3}{4}$. **11.** $\frac{23}{40}$. **12.** $\frac{3}{40}$. **13.** $\frac{1}{80}$. **14.** $\frac{1}{40}$. **15.** $\frac{7}{400}$. **16.** $\frac{1}{160}$. **17.** $\frac{3}{800}$.—**2.** .25. **3.** .125. **4.** .625. **5.** .875. **6.** .3125. **7.** .4375. **8.** .55. **9.** .95. **10.** .52. **11.** .76. **12.** .65625. **13.** .84375. **14.** .83$\frac{1}{3}$. **15.** .53$\frac{1}{3}$. **16.** .81$\frac{9}{11}$. **17.** .346$\frac{2}{3}$.

Page 187.—2. 189.38. **3.** 287.84. **4.** 497.935. **5.** 1605.50494. **6.** 10611.106843. **7.** 254.507. **8.** 59.3814. **9.** 178.33262. **10.** 228.0974. **11.** 1783.71679. **12.** 225.5081355. **13.** 40.792 cords.

Page 188.—14. 224.75 miles.—**2.** 37.91. **3.** 23.17. **4.** 47.32. **5.** 79.942. **6.** 2.741. **7.** 2.647. **8.** 191.2916. **9.** 4.2397. **10.** .999. **11.** 1.818. **12.** .2992. **13.** 6.299293.

Page 189.—14. 164.75 barrels.—**2.** 2.7306. **3.** 6.545. **4.** 18.711. **5.** 84.8988. **6.** 341.4775. **7.** 2468.9488. **8.** 2991.6705. **9.** 7672.47. **10.** 16751.9256. **11.** .000229191. **12.** .003309189. **13.** .000286902. **14.** 117.5625 miles. **15.** Sum by .75390625. **16.** 2296.4375 gallons.

Page 190.—2. 3.1416. **3.** 3.1416. **4.** 18.25. **5.** 21. **6.** 4680. **7.** 3500. **8.** .67. **9.** 1600. **10.** .7854. **11.** 10.75. **12.** 13.6$\frac{1}{19}$. **14.** 11.26$\frac{8}{17}$.

Page 192.—1. $6.25. **2.** $25.36. **3.** $8.456. **4.** $20.752. **5.** $67.84. **6.** $4.67. **7.** $25.058. **8.** $358.065.

Page 193.—2. $193.78. **3.** $272.22. **4.** $237.66. **5.** $224.529. **6.** $838.87. **7.** $367.34. **8.** $133.70. **9.** $126.75. **10.** $2594.24.

Page 194.—2. $65.09. **3.** $34.24. **4.** $68.29. **5.** $52.47. **6.** $80.48. **7.** $237.08. **8.** $6.25. **9.** $13.54. **10.** $359.75. **11.** $519.75. **12.** $53.75. **13.** $88.05.

Page 195.—14. $1790.63. **15.** $48.45.—**2.** $104.56. **3.** $186.35. **4.** $342.72. **5.** $300.16. **6.** $714.15. **7.** $579. **8.** $939.51. **9.** $1283.70. **10.** $2970.56. **11.** $3464.72. **12.** $16.25. **13.** $1809. **14.** $978.75. **15.** $406.25.

Page 196.—16. $2298.64. **17.** $178.60. **18.** $12238.85. **19.** $5313.75.—**2.** $1.45. **3.** $2.87. **4.** $2.31. **5.** $5.25. **6.** $21.60. **7.** $542. **8.** $5.40.

Page 197.—9. $109.65. **10.** $8.50. **11.** $142.80. **12.** $789.60. **13.** $3187.50. **14.** $79.56.—**2.** 600. **3.** 182. **4.** 325. **5.** 5. **6.** 6. **7.** 50. **8.** 78 oxen. **9.** 37 pigs. **10.** 105 days. **11.** 48 horses. **12.** 96 cords. **13.** 365 days. **14.** 16 pieces.

Page 198.—1. $47.09.

Page 199.—2. $285.50. **3.** $463.10. **4.** $12.90.

Page 200.—5. $2.50.—**1.** $3.50. **2.** $17.18. **3.** $159.65. **4.** Thomas, $5.75.

Page 202.—2. 354 pwt. **3.** 202 oz.; 296 oz. **5.** 11 pwt. 11 gr. **6.** 1 lb. 3 oz. **7.** 1330 pwt.

Page 203.—1. 175 oz. **2.** 1675 lb. **3.** 8 lb. 7 oz. **4.** 197 cwt. **5.** 7 cwt. 24 lb. **6.** 1 cwt. 6 lb. 7 oz. **7.** 1 T. 10 cwt.

Page 204.—1. 500 gr.; 158 gr. **2.** 90 ℈; 46 ℈. **3.** 216 ʒ; 203 ʒ. **4.** 13 ℈ 17 gr. **5.** 38 ʒ 6 ʒ. **6.** 1 ℔ 4 ʒ.

Page 205.—1. 35 pt.; 45 pt. **2.** 135 qt.; 162 qt. **3.** 19 gal. 2 qt.; 23 gal. 1 pt. **4.** 15 gal.; 20 gal.; 32 gal. 1 pt. **5.** 56 qt.; 18 qt.

Page 206.—1. 35 pt. **2.** 61 qt. **3.** 6 pk. 7 qt.; 6 pk. **4.** 9 pk. 1 pt.; 16 pk. 1 pt. **5.** 4 bu. 3 pk. 5 qt.; 6 bu. 1 pk. 3 qt. **6.** 1 bu. 3 pk. 4 qt.; 5 bu. 1 pk. 5 qt. 1 pt.

Page 207.—1. 114 in.; 152 in. **2.** 81 ft.; 177 ft. **3.** 4180 yd.; 32 yd. 1 ft. **4.** 10 ft. 5 in.; 11 ft. 10 in. **5.** 24 yd.; 4 yd. 1 ft.; 7 yd. 1 ft. 11 in. **6.** 2 mi. 4 rd.; 13 rd. 3½ yd.

Page 208.—1. 720 sq. in. **2.** 108 sq. ft. **3.** 242 sq. yd. **4.** 12 sq. yd. **5.** 12 sq. ft. **6.** 6 A.; 7 A. 130 sq. rd. **7.** 24 P. **8.** 16 A.

Page 209.—1. 216 cu. ft.; 20 cu. yd.

Page 210.—2. 768 cu. ft.; 96 cd. ft. **3.** 12192 cu. in. **4.** 4 cu. ft. 1557 cu. in.—**1.** 120 sec.; 180 sec.; 240 sec. **2.** 120 min.; 180 min.; 240 min.; 48 h.; 72 h. **3.** 38 da.; 47 da.; 1095 da.; 3 da. **4.** 6 da.; 9 da.; 12 da. 12 h. **5.** 10 yr.; 100 yr.

Page 212.—1. 240″; 300″; 1440″. **2.** 4′; 8′ 20″. **3.** 300′; 324′; 996′. **4.** 6°; 10° 50′.

Page 213.—1. 144; 1728. **2.** 96 sheets; 82 sheets. **3.** 5 qu.; 90 qu.; 19200 qu. **4.** 1920 sheets; 130 qu.; 1536 sheets.

Page 214.—2. 1820 gr. **3.** 8946 gr. **4.** 56252 oz. **5.** 27018 lb. **6.** 7192 gr. **7.** 62415 gr. **8.** 135 pt. **9.** 255 pt. **10.** 653 pt. **11.** 210 in. **12.** 4903 in. **13.** 813 gi. **14.** 443 pt. **15.** 523244 in. **16.** 30584 yd. **17.** 86 sq. yd. **18.** 2445 sq. ft. **19.** 32083 sq. yd. **20.** 1347928 cu. in. **21.** 10605 min. **22.** 828 h. **23.** 67095 h.

Page 215.—3. 3 oz. 15 pwt. 20 gr. **4.** 6 oz. 15 pwt. 8 gr. **5.** 5 cwt. 59 lb. 2 oz. **6.** 3 cwt. 53 lb. 4 oz. **7.** 3 T. 10 cwt. 18 lb. **8.** 1 cwt. 23 lb. 2 oz. **9.** 4 ℥ 7 ʒ 2 ℈ 12 gr. **10.** 1 ℔ 1 ℥ 5 gr. **11.** 168 gal. 3 qt. **12.** 50 gal. 3 qt. 1 pt. 2 gi. **13.** 56 gal. 2 qt. 1 pt. 1 gi. **14.** 17 bu. 3 qt. 1 pt. **15.** 22 bu 2 pk. 1 qt. 1 pt. **16.** 41 bu. 1 pk. 6 qt. 1 pt. **17.** 116 yd. 2 ft. 10 in. **18.** 24 rd. 4 yd. 7 in. **19.** 35 rd. 1 ft. 6 in. **20.** 1 mi. 167 rd. 4 yd. 1 ft. 6 in. **21.** 4 sq. yd. 7 sq. ft. 123 sq. in. **22.** 32 P. 28 sq. yd. **23.** 1 A. 29 sq. yd. **24.** 1 cu. ft. 1292 cu. in. **25.** 77 cd. 12 cu. ft. **26.** 158 cd. ft. **27.** 1 h. 12 min. 25 sec. **28.** 3 da. 10 h. 51 min. **29.** 1 yr. 6 da. 17 h. **30.** 2° 25′ 30″. **31.** 1 ℔ 1 ℥ 6 ʒ 1 ℈ 18 gr. **32.** 4 R. 17 qu. 12 sheets.

Page 216.—2. 104 gal. 3 qt. **3.** 20 bu. 3 pk. 2 qt. 1 pt. **4.** 60 bu. 2 qt. **5.** 174 lb. 3 oz. **6.** 127° 7′ 27″. **7.** 229 lb. 8 oz. 19 pwt. 16 gr. **8.** 88 rd. 5 yd. 1 ft. 6 in. **9.** 61 T. 11 cwt. 9 lb. 14 oz. **10.** 4 da. 16 h. 44 min. 45 sec.

Page 217.—2. 17 gal. 3 qt. **3.** 2 lb. 10 oz. 19 pwt. **4.** 8 bu. 2 pk. 6 qt. 1 pt. **5.** 4 lb. 11 oz. 3 pwt. 21 gr. **6.** 2 T. 15 cwt. 56 lb. 14 oz. **7.** 23 h. 29 min. 45 sec. **8.** 31 mi. 317 rd. 4 yd. 1 ft. 6 in. **9.** 92 yr. 11 mo. 3 wk. 2 da. 20 h.—**Art. 166. 2.** 50 lb. 2 oz. 5 pwt. **3.** 179 gal. 1 qt. 1 pt. **4.** 51 bu. 2 pk. **5.** 113 gal. 1 qt. **6.** 126 lb. 10 oz. 8 pwt. **7.** 142 bu. 1 qt. **8.** 34 yd. 1 ft. 4 in. **9.** 61 yr. 5 mo. 24 da. **10.** 119 wk. 2 da. **11.** 4 da. 12 h. 53 min. **12.** 203 ℔ 10 ℥. **13.** 268 S. 6° 30′ 40″.

Page 218.—14. 3 T. 19 cwt. 2 qr. **15.** 351 mi. 240 rd.—**2.** 16 lb. 8 oz. 15 pwt. **3.** 35 gal. 1 qt. 1 pt. **4.** 12 bu. 2 pk. 4 qt. **5.** 18 gal. 3 qt. 1 pt. **6.** 6 yd. 2 ft. 8 in. **7.** 15 lb. 10 oz. 6 pwt. **8.** 8 yr. 9 mo. 12 da. **9.** 9 wk. 5 da. 13 h. **10.** 16 T. 38 lb. **11.** 22 A. 92 P. 29 sq. yd. **12.** 31 mi. 129⅔ rd. **13.** 4 lb. 2 oz. 10 pwt. 20⅘ gr. **14.** 9. **15.** 6. **16.** 5.

Page 219.—1. 14 h. **2.** 30 h. **3.** 29 h. **4.** 38 h. 15 min. **5.** 59 h. 50 min. **6.** 8 h. **7.** $18. **8.** 32 da.; 46 da. **10.** 88 da. **11.** 109 da. **12.** 117 da. **13.** 143 da. **14.** 107 da. **15.** 180 da. **16.** 239 da. **17.** 295 da.

Page 220.—2. 1 yr. 9 mo. 5 da. **3.** 5 yr. 8 mo. 15 da. **4.** 2 yr. 9 mo. 27 da. **5.** 1 yr. 4 mo. 17 da. **6.** 1 yr. 16 da. **7.** 1 yr. 5 mo. 24 da. **8.** 1 yr. 2 mo. 27 da. **9.** 7 yr. 9 mo. 1 da.—*Ages.* **1.** 67 yr. 9 mo. 22 da. **2.** 90 yr. 8 mo. 4 da. **3.** 83 yr. 3 mo. 2 da. **4.** 85 yr. 3 mo. 12 da.

Page 221.—5. 73 yr. 2 mo. 6 da. **6.** 80 yr. 7 mo. 12 da. **7.** 78 yr. 2 mo. 23 da. **8.** 79 yr. 7 mo. 19 da. **9.** 68 yr. 1 mo. 25 da. **10.** 71 yr. 9 mo. 19 da. **11.** 53 yr. 7 mo. 13 da. **12.** 65 yr. 7 mo. 15 da. **13.** 74 yr. 1 mo. **14.** 64 yr. 10 mo. 15 da. **15.** 77 yr. 1 mo. 6 da. **16.** 56 yr. 2 mo. 3 da. **17.** 66 yr. 7 mo. 2 da. **18.** 63 yr. 2 mo. 26 da. **19.** 70 yr. 3 mo. 13 da. **20.** 49 yr. 10 mo. **21.** 56 yr. 1 mo. 13 da.

Page 222.—1. 108000 times. **2.** 13 da. 10 h. 40 min. **3.** 6250 h. **4.** 86400 times. **5.** $2. **6.** 3600. **7.** 16 vials. **8.** 5259600 min. **9.** 40 doses. **10.** 384. **11.** 384 sq. rd. **12.** 45 sq. ft. **13.** 36 cu. ft. **14.** $2160. **15.** 6 cd. **16.** 4 cd.

Page 223.—17. $13.50. **18.** 1 year. **19.** $1.28. **20.** $337.50. **21.** $20. **22.** $15.36. **23.** $983.04. **24.** $1.98. **25.** 4400 steps. **26.** $336. **27.** $850. **28.** $120. **29.** 80 A. **30.** $70.56. **31.** $34.56. **32.** 1 ℔ 6 ℥ 3 ʒ. **33.** 10 cents.

Page 224.—34. 32 bottles. **35.** $150. **36.** 66 baskets. **37.** 4 ℥. **38.** 220 pills. **39.** $60. **40.** 231 panels. **41.** $11.28. **42.** 1284 sheets. **43.** $800. **44.** 43 bu. **45.** 1 bu. 3 pk. $2\frac{2}{3}$ qt. **46.** $2\frac{1}{2}$ mi. **47.** $78.75. **48.** $240.

Page 225.—2. 12; 12; 32; 8; 24; 15; $10\frac{6}{25}$. **3.** $3\frac{3}{4}$; 16; 18; 40; 18; 8; 18. **4.** $30. **5.** $90. **6.** 96 pupils. **8.** 90; 64; 300; 48; 30. **9.** 150.

Page 226.—11. 50%, 25%; 20%; $12\frac{1}{2}$%; $33\frac{1}{3}$%. **12.** 25%.—**2.** $14. **3.** $11.40. **4.** 10 yd. **5.** 32.76 lb. **6.** 450.1 h. **7.** 986.8 lb. **8.** 40 oz. **9.** 105 yd. **10.** 360 bu. **11.** $46.40. **12.** 315 A. **13.** $5520. **14.** 486.40 gal. **15.** $2000. **16.** 80%. **17.** $303\frac{3}{19}$. **18.** $16\frac{2}{3}$%. **19.** $600.

Page 227.—2. $90. **3.** $166.80. **4.** $8.352. **5.** $10.15. **6.** $30.3408. **7.** $45.132. **8.** $44.10. **9.** $216.20. **10.** $49.45. **11.** $102.40.

Page 228.—2. $20.64. **3.** $6.144. **4.** $2.116. **5.** $83.5741. **6.** $6.787+. **7.** $17.26+. **8.** $15.96+. **9.** $54.

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